Station Keeping Strategies for Solar Sails

A. Farrés[1] & À. Jorba[2]

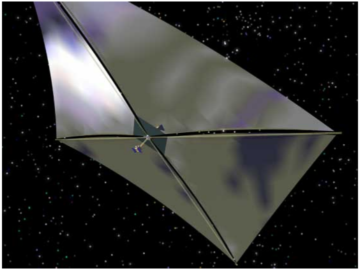
[1]Institut de Mécanique Céleste et de Calcul des Éphémérides,
Observatoire de Paris
(afarres@imcce.fr)

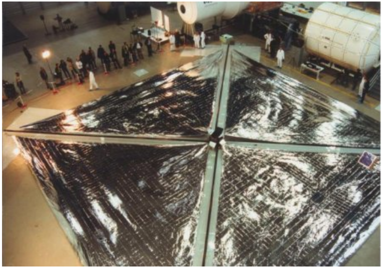
[2]Departament de Matemàtica Aplicada i Ànalisi,
Universitat de Barcelona
(angel@maia.ub.es)

Background on Solar Sails

What is a Solar Sail ?

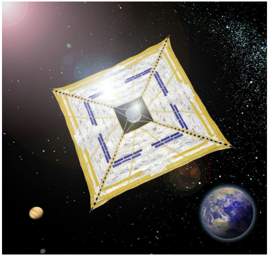
- Solar Sails a proposed form of propulsion system that takes advantage of the Solar radiation pressure to propel a spacecraft.
- The impact of the photons emitted by the Sun on the surface of the sail and its further reflection produce momentum on it.
- Solar Sails open a wide new range of possible missions that are not accessible by a traditional spacecraft.

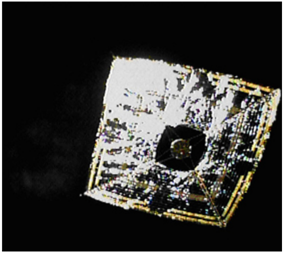




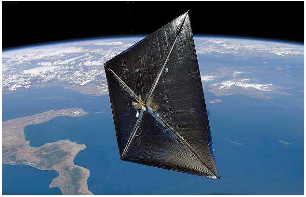
There have recently been two successful deployments of solar sails in space.

- IKAROS: in June 2010, JAXA managed to deploy the first solar sail in space.



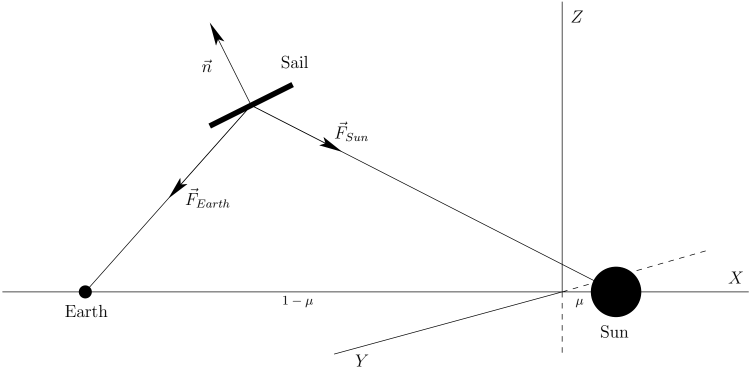


- NanoSail-D2: in January 2011, NASA deployed the first solar sail that would orbit around the Earth.



The Dynamical Model

We use the Restricted Three Body Problem (RTBP) taking the Sun and Earth as primaries and including the solar radiation pressure due to the solar sail.



The Solar Sail

We consider the solar sail to be flat and perfectly reflecting. Hence, the force due to the solar radiation pressure is in the normal direction to the surface of the sail.

The force due to the sail is defined by the *sail's orientation* and the *sail's lightness number*.

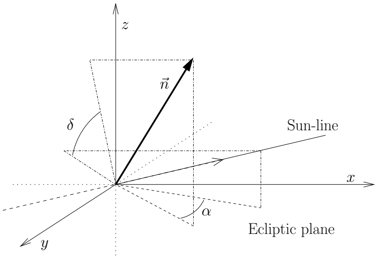
- The *sail's orientation* is given by the normal vector to the surface of the sail, $\vec{n}$. It is parametrised by two angles, α and δ.
- The *sail's lightness number* is given in terms of the dimensionless parameter β. It measures the effectiveness of the sail.

Hence, the force is given by:

$$\vec{F}_{sail} = \beta \frac{m_s}{r_{ps}^2} \langle \vec{r}_s, \vec{n} \rangle^2 \vec{n}.$$

The Sail Orientation

There are several ways to define the two angles that parametrise the sail orientation α and δ.



We define:

- α is the angle between the projection of $\vec{r}_s$ and $\vec{n}$ on the ecliptic plane.
- δ is the difference between: *a*) the angle of the $\vec{r}_s$ with the ecliptic plane; and *b*) the angle of $\vec{n}$ with the ecliptic plane.
- as the sail cannot point towards the Sun, we have that $\langle \vec{r}_s, \vec{n} \rangle \geq 0$.

Equations of Motion

The equations of motion are:

$$
\begin{aligned}
\ddot{x} &= 2\dot{y} + x - (1-\mu)\frac{x-\mu}{r_{ps}^3} - \mu\frac{x+1-\mu}{r_{pe}^3} + \beta\frac{1-\mu}{r_{ps}^2}\langle\vec{r}_s,\vec{n}\rangle^2 n_x, \\
\ddot{y} &= -2\dot{x} + y - \left(\frac{1-\mu}{r_{ps}^3} + \frac{\mu}{r_{pe}^3}\right) y + \beta\frac{1-\mu}{r_{ps}^2}\langle\vec{r}_s,\vec{n}\rangle^2 n_y, \\
\ddot{z} &= -\left(\frac{1-\mu}{r_{ps}^3} + \frac{\mu}{r_{pe}^3}\right) z + \beta\frac{1-\mu}{r_{ps}^2}\langle\vec{r}_s,\vec{n}\rangle^2 n_z,
\end{aligned}
$$

where $\vec{n} = (n_x, n_y, n_z)$ is the normal direction to the surface of the sail with,

$$
\begin{aligned}
n_x &= \cos(\phi(x,y)+\alpha)\cos(\psi(x,y,z)+\delta), \\
n_y &= \sin(\phi(x,y,z)+\alpha)\cos(\psi(x,y,z)+\delta), \\
n_z &= \sin(\psi(x,y,z)+\delta),
\end{aligned}
$$

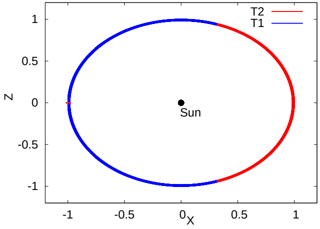
and $\vec{r}_s = (x-\mu, y, z)/r_{ps}$ is the Sun - sail direction.

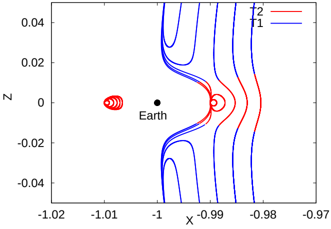
Equilibrium Points (I)

- The RTBP has 5 equilibrium points (L_i). For small β, these 5 points are replaced by 5 continuous families of equilibria, parametrised by α and δ.
- For a fixed small value of β, we have 5 disconnected family of equilibria around the classical L_i.
- For a fixed and larger β, these families merge into each other. We end up having two disconnected surfaces, S_1 and S_2. Where S_1 is like a sphere and S_2 is like a torus around the Sun.
- All these families can be computed numerically by means of a continuation method.

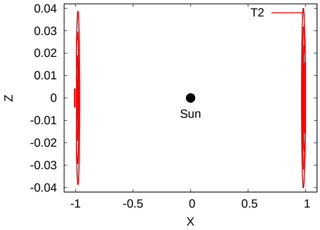
Equilibrium Points (II)

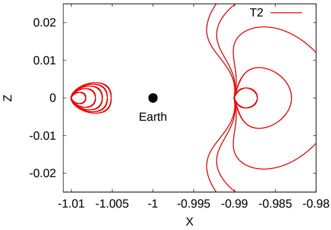
Equilibrium points in the XY plane





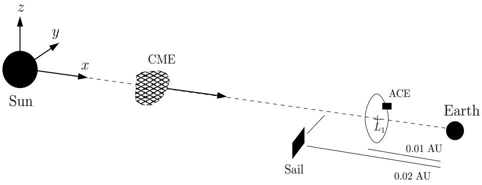
Equilibrium points in the XZ plane



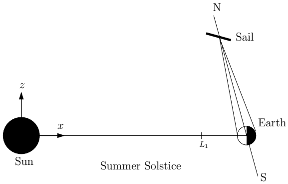


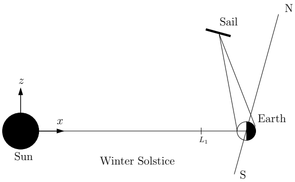
Interesting Missions Applications

Observations of the Sun provide information of the geomagnetic storms, as in the Geostorm Warning Mission.



Observations of the Earth's poles, as in the Polar Observer.

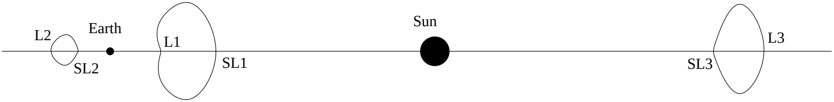




Periodic Motion Around Equilibria

We must add a constrain on the sail orientation to find bounded motion. One can see that when $\alpha = 0$ and $\delta \in [-\pi/2, \pi/2]$ (i.e. only move the sail vertically w.r.t. the Sun - sail line):

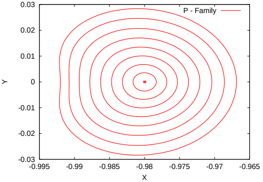
- There are 5 disconnected families of equilibrium points parametrised by δ, we call them $FL_{1,\dots,5}$ (each one related to one of the Lagrangian points $L_{1,\dots,5}$).
- Three of these families ($FL_{1,2,3}$) lie on the $Y = 0$ plane, and the linear behaviour around them is of the type saddle$\times$centre$\times$centre.
- We consider the sail orientation to be fixed along time.

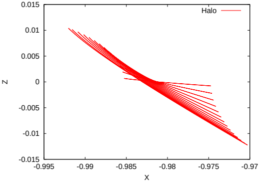


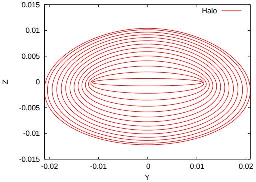
(Schematic representation of the equilibrium points on $Y = 0$)

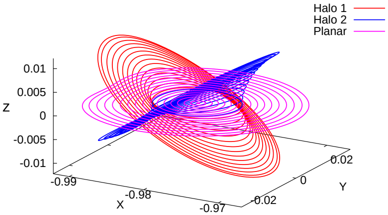
$\mathcal{P}$-Family of Periodic Orbits

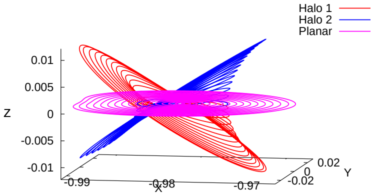
Periodic Orbits for $\alpha = 0, \delta = 0$.







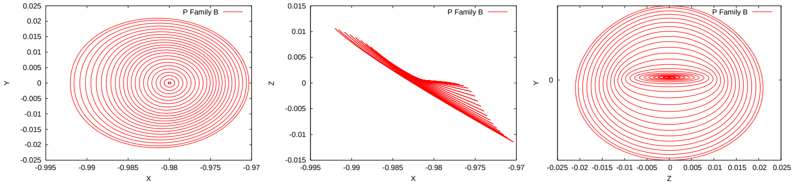




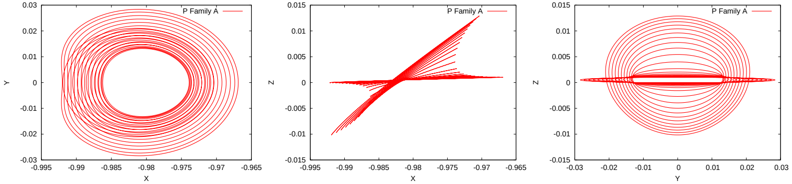
$\mathcal{P}$-Family of Periodic Orbits

Periodic Orbits for $\alpha = 0, \delta = 0.01$.

Main family of periodic orbits for $\delta = 0.01$

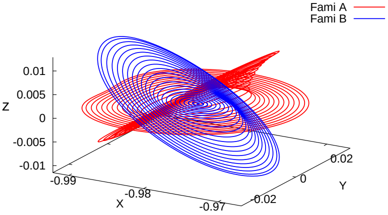


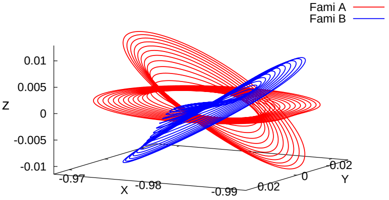
Secondary family of periodic orbits for $\delta = 0.01$

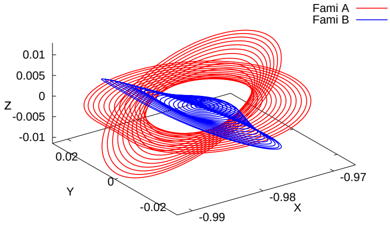


$\mathcal{P}$-Family of Periodic Orbits

Periodic Orbits for $\alpha = 0, \delta = 0.01$.

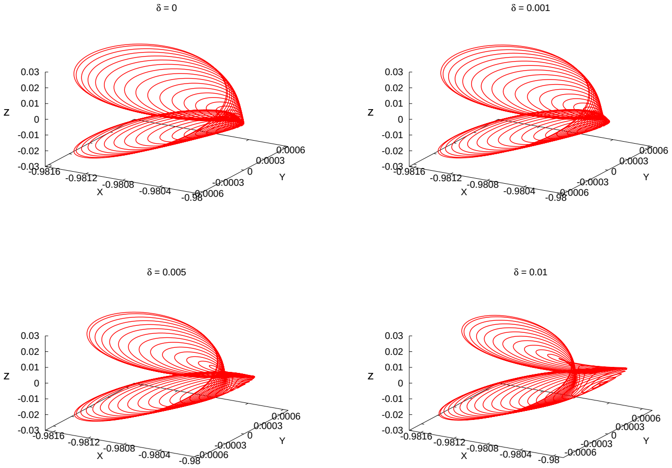






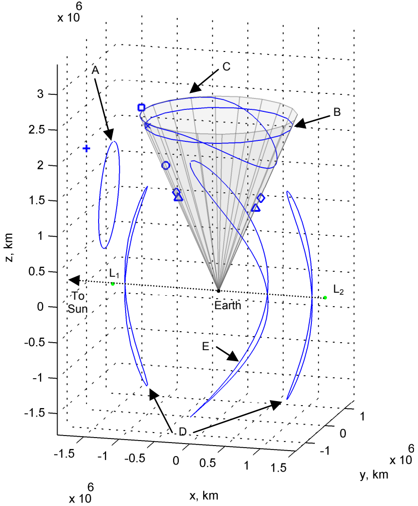
$\mathcal{V}$-Family of Periodic Orbits

Periodic Orbits for $\alpha = 0, \delta = 0, 0.001, 0.005, 0.01$.



Interesting Mission Applications

Observations of the Earth's poles, as in the Polar Observer.



AIM of this TALK

We want to design Station Keeping Strategy to maintain the trajectory of a solar sail close to an unstable equilibrium point.

Instead of using *Control Theory Algorithms*, we want to use *Dynamical System Tools* to find a station keeping algorithm for a Solar Sail.

AIM of this TALK

We want to design Station Keeping Strategy to maintain the trajectory of a solar sail close to an unstable equilibrium point.

Instead of using *Control Theory Algorithms*, we want to use *Dynamical System Tools* to find a station keeping algorithm for a Solar Sail.

The main ideas are ...

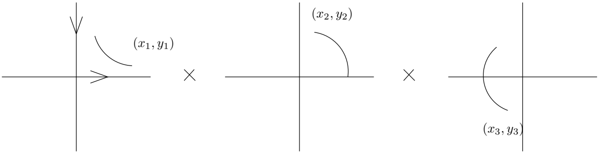
- To focus on the linear dynamics around an equilibrium point or periodic orbit and study how this one varies when we change the sail orientation.
- Find changes in the sail orientation (i.e. the phase space) to make the system act in our favour: keep the trajectory close to a given equilibrium point or periodic orbit.

Station Keeping Strategies Around Equilibria

Station Keeping for a Solar Sail

We focus on the two previous missions, where the equilibrium points are unstable with two real eigenvalues, $\lambda_1 > 0, \lambda_2 < 0$, and two pair of complex eigenvalues, $\nu_{1,2} \pm \mathrm{i}\omega_{1,2}$, with $|\nu_{1,2}| << |\lambda_{1,2}|$.

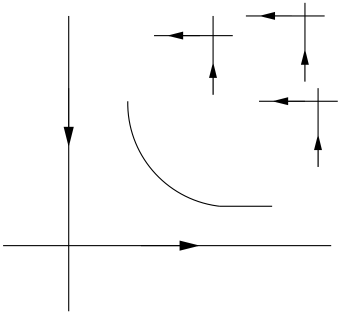
- To start we can consider that the dynamics close the equilibrium point is of the type saddle $\times$ centre $\times$ centre.
- From now on we describe the trajectory of the sail in three reference planes defined by each of the eigendirections.



- For small variations of the sail orientation, the equilibrium point, eigenvalues and eigendirections have a small variation. We will describe the effects of the changes on the sail orientation on each of these three reference planes.

Schematic Idea of the Station Keeping Strategy (I)

In the saddle projection of the trajectory:

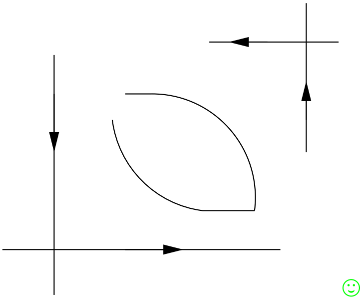


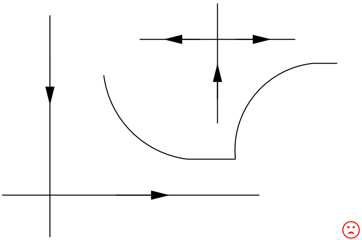
- When we are close to the equilibrium point, p_0, the trajectory escapes along the unstable direction.
- When we change the sail orientation the position of the equilibrium point is shifted and its eigendirections vary slightly.

Schematic Idea of the Station Keeping Strategy (II)

In the saddle projection of the trajectory:

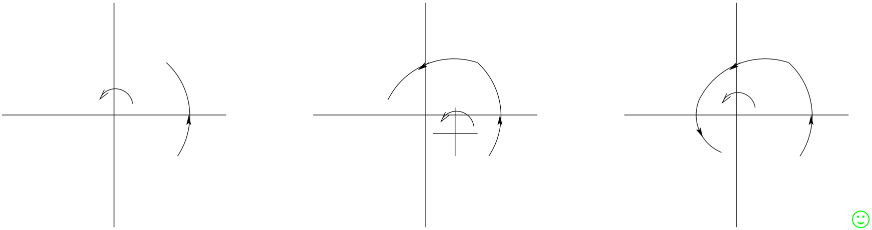
- Now the trajectory will escape along the new unstable direction.
- We want to find a new sail orientation (α, δ) so that the trajectory will come close to the stable direction of p_0.





Schematic Idea of the Station Keeping Strategy (III)

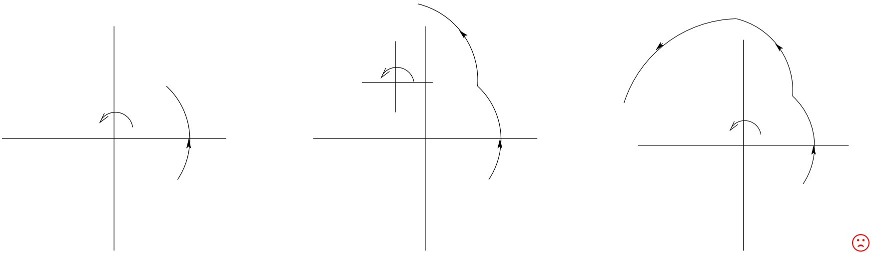
In the centre projection of the trajectory:



A sequence of changes on the sail orientation implies a sequence of rotations around different equilibrium points on the centre projection, which can result of an unbounded growth.

Schematic Idea of the Station Keeping Strategy (III)

In the centre projection of the trajectory:



A sequence of changes on the sail orientation implies a sequence of rotations around different equilibrium points on the centre projection, which can result of an unbounded growth.

Schematic idea of the Station Keeping Algorithm

We look at the sails trajectory in the reference system $\{x_0; \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5, \vec{v}_6\}$, so $z(t) = x_0 + \Sigma_i s_i(t)\vec{v}_i$.

During the station keeping algorithm:

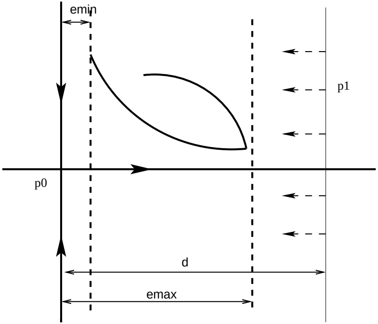
1. when $\alpha = \alpha_0, \delta = \delta_0$: if $|s_1(t)| \geq \varepsilon_{max}$ $\Rightarrow$ choose new sail orientation $\alpha = \alpha_1, \delta = \delta_1$.
2. when $\alpha = \alpha_1, \delta = \delta_1$: if $|s_1(t)|$ small $\Rightarrow$ restore the sail orientation: $\alpha = \alpha_0, \delta = \delta_0$.
3. REPEAT

1st Idea for finding $\alpha_{new}, \delta_{new}$

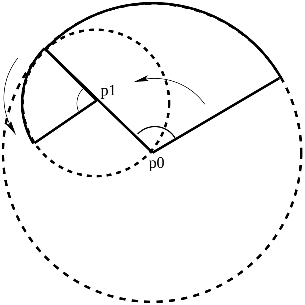
We will choose a the position of the new equilibrium point (i.e. a new sail orientation) so that projection of the trajectory on the saddle will come back and the two centre projections remain bounded ?

1st Idea for finding $\alpha_{new}, \delta_{new}$

We will choose a the position of the new equilibrium point (i.e. a new sail orientation) so that projection of the trajectory on the saddle will come back and the two centre projections remain bounded ?



×



The constants ε_{min}, ε_{max} and d will depend on the mission requirements and the dynamics around the equilibrium point.

1st Idea for finding $\alpha_{new}, \delta_{new}$

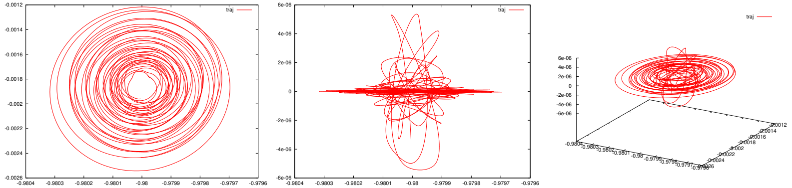
- We do not know explicitly the position of the equilibrium points $p(\alpha, \delta)$. But we can compute the linear approximation of this function:

$$p(\alpha, \delta) = p(\alpha_0, \delta_0) + Dp(\alpha_0, \delta_0) \cdot (\alpha - \alpha_0, \delta - \delta_0)^T.$$

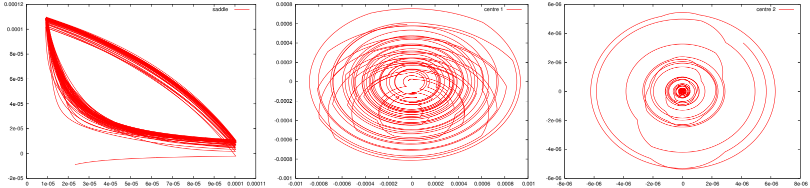
- There are some restrictions of the position of the new equilibria when we change α and δ. We have 2 unknowns and at least 6 conditions that must be satisfied.
- We will change the sail orientation so that the position of the new fixed point is as close as possible to the desired new equilibrium point and in the correct side in the saddle projection.
- To decide the new sail orientation we will assume that the eigenvalues and eigendirections do not vary when the sail orientation is changed.

Results for the Geostorm Mission (RTBPS)

XY and XZ and XYZ Projections

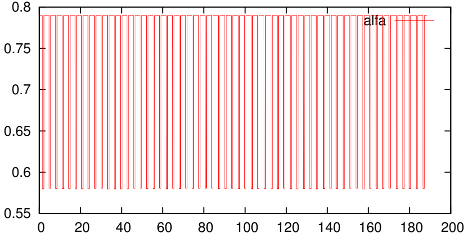


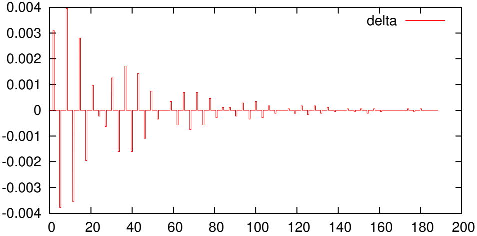
Saddle × Centre × Centre Projections



Results for the Geostorm Mission (RTBPS)

Variation of the sail orientation





2st Idea for finding $\alpha_{new}, \delta_{new}$

The computation of variational equations (of suitable order) w.r.t. α and δ gives explicit expressions for the effect of different orientations (close to the reference values $\alpha = \alpha_0, \delta = \delta_0$) trajectory.

$$\phi_t(x_0, \alpha_0 + h_a, \delta_0 + h_d) = \phi_t(x_0, \alpha_0, \delta_0) + \frac{\partial \phi}{\partial \alpha}(x_0, \alpha_0, \delta_0) \cdot h_a + \frac{\partial \phi}{\partial \delta}(x_0, \alpha_0, \delta_0) \cdot h_d,$$

With this we can impose conditions on the "future" of the orbit and find orientations that fulfil them (or show that the condition is unattainable).

- We will define the parameters ε_{max}, Dt_{min} and Dt_{max} that will vary for each mission application.
- We will find $\alpha_{new}, \delta_{new}$ and $dt \in [Dt_{min}, Dt_{max}]$ so that the trajectory is close to the fixed point.

2st Idea for finding $\alpha_{new}, \delta_{new}$

We proceed as follows:

- We use:

$$\mathcal{F}(dt, h_a, h_d) = \phi_{dt}(t_0, x_0, \alpha_0, \delta_0) + \frac{\partial \phi}{\partial \alpha}(t_0, x_0, \alpha_0, \delta_0) \cdot h_a + \frac{\partial \phi}{\partial \delta}(t_0, x_0, \alpha_0, \delta_0) \cdot h_d,$$

  to find dt, h_a and h_d so that the trajectory at time $t_0 + dt$ is close to the nominal orbit.

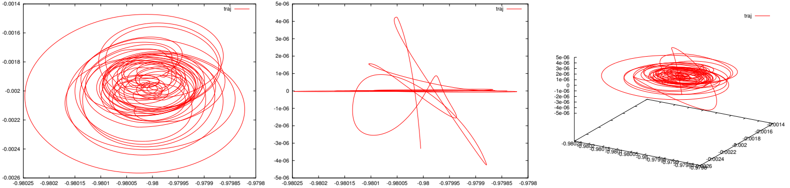
- For each $dt \in [Dt_{min}, Dt_{max}]$ we will find h_a and h_d such that $s_1 = 0$ and (s_5, s_6) are minimum (i.e. we are close to stable direction and one of the centre projections is small).

- From all the dt, h_a and h_d we chose the one such that the other centre projection (s_3, s_4) is minimised.

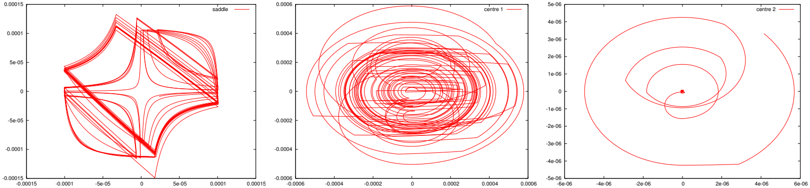
$$\alpha_{new} = \alpha_0 + h_a^*, \qquad \delta_{new} = \delta_0 + h_d^*$$

Results for the Geostorm Mission (RTBPS)

XY and XZ and XYZ Projections

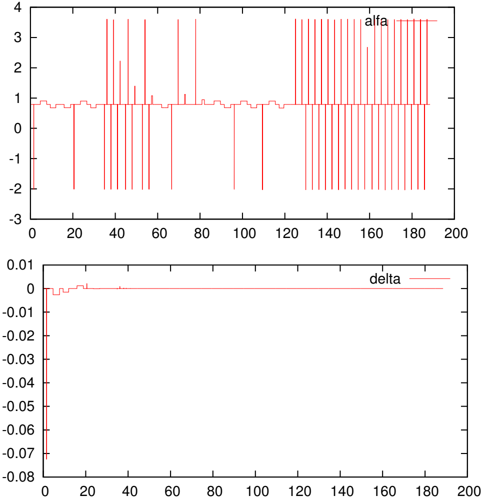


Saddle × Centre × Centre Projections



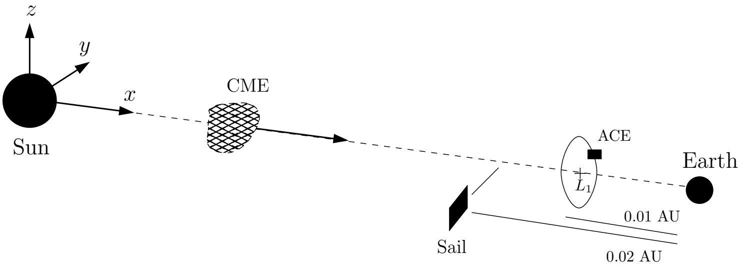
Results for the Geostorm Mission (RTBPS)

Variation of the sail orientation



Results for the Geostorm Mission

We have applied these station keeping strategy to different mission scenarios. We show the results for the Geostorm Warning Mission.



Mission Parametres:

- In the RTBPS to have the appropriate fixed point we take: $\beta = 0.051689$ ($a_0 \approx 0.3$ mm/s^2), $\alpha_0 = 0.7897°$ and $\delta_0 = 0°$.

Robustness of the Method

Mission Parametres:

- Alg. FP: We have taken $\varepsilon_{max} = 5 \cdot 10^{-5}$AU (the escape distance), $\varepsilon_{min} = 10^{-6}$AU (minimum distance), $d = 2$ (estimate position of the new fixed point in the saddle projection).
- Alg VF: We have taken $\varepsilon_{max} = 5 \cdot 10^{-5}$AU (the escape distance), $dt_{min} = 2$ days and $dt_{max} = 169$ days (the minimum and maximum time between manoeuvres).

Test:

- We have done a Monte Carlo Simulation taking a 1000 random initial conditions and applied the station keeping strategy for 20 years.
- We have tested the robustness of these strategies against different sources of errors. We have considered errors on the position and velocity determination, as well as errors in the sail orientation.

Note : All the simulations have been done using the full set of equations, we only use the

Results for the Geostorm Mission (RTBPS)

Algorithm Used: **Fixed Point Algorithm**

EType	% Succ.	Δt (days)	$\Delta\alpha$ (deg)	$\Delta\delta$ (deg)
E0	100.0 %	158.32 - 38.90	0.211 - 0.209	4.884e-03 - 5.040e-06
V1	100.0 %	363.40 - 35.38	0.228 - 0.189	5.588e-02 - 2.179e-04
V2	79.0 %	370.85 - 28.23	0.283 - 0.101	2.123e-01 - 6.749e-04

EType stands for the kind of errors considered in each simulation: E0 = No errors, V1, V2 = Errors on Position, Velocity and Sail Orientation, where V1 = 0.01°, V2 = 0.05°

Results for the Geostorm Mission (RTBPS)

Algorithm Used: **Variational Equation Algorithm**

EType	% Succ.	Δt (days)	$\Delta\alpha$ (deg)	$\Delta\delta$ (deg)
E0	100.0 %	317.88 - 2.32	2.82 - 0.109	0.160 - 0.000
V1	100.0 %	361.96 - 2.32	4.09 - 0.098	0.557 - 2.38e-04
V2	100.0 %	334.56 - 2.32	4.47 - 0.041	2.598 - 4.54e-04

EType stands for the kind of errors considered in each simulation: E0 = No errors, V1, V2 = Errors on Position, Velocity and Sail Orientation, where V1 = 0.01°, V2 = 0.05°

Results for the Geostorm Mission (RTBPS)

Variation of the sail orientation

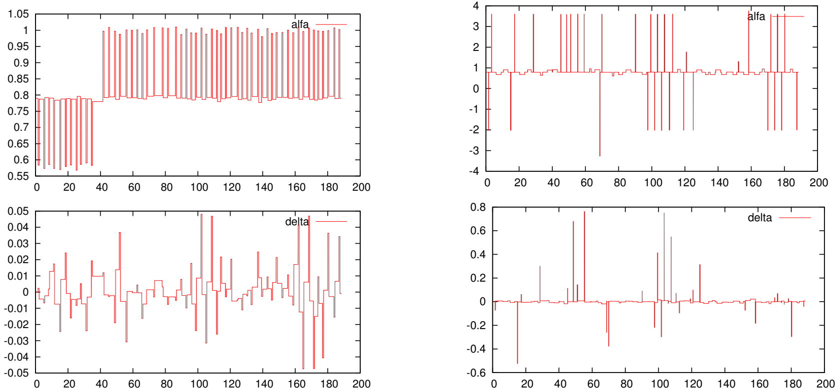
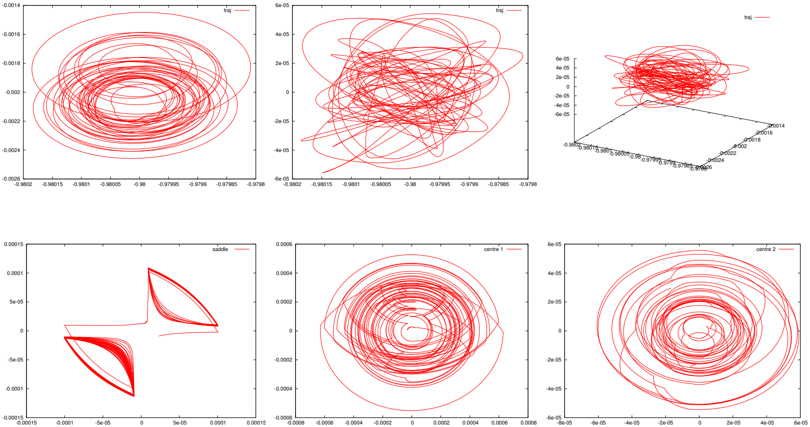


Figure: Fixed Point Alg.(left) Variational Flow Alg.(right)

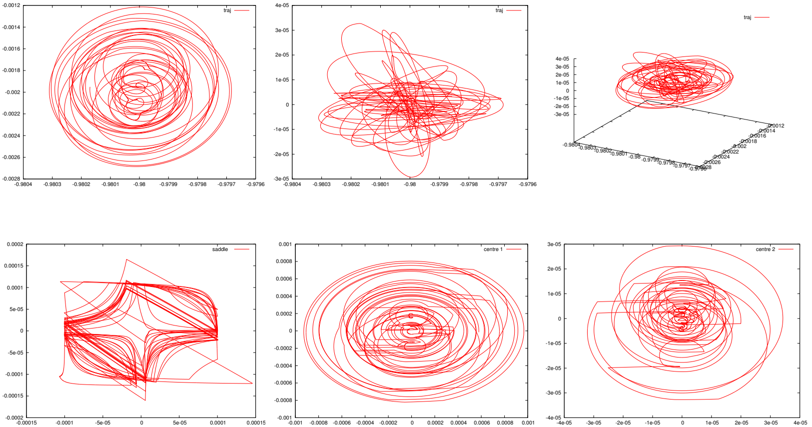
Results for the Geostorm Mission (RTBPS)

Algorithm Used: **Fixed Point Algorithm** (simulations with errors).



Results for the Geostorm Mission (RTBPS)

Algorithm Used: **Variational Equations Algorithm** (simulations with errors).



Station Keeping Strategies Around Periodic Orbits

Linear dynamics around a Periodic Orbit (I)

For the Periodic Orbits that we considered in this study the eigenvalues $(\lambda_{1,\dots,6})$ of the monodromy matrix satisfy: $\lambda_1 > 1, \lambda_2 < 1,\ \lambda_3 = \bar{\lambda}_4$ are complex with modulus 1 and $\lambda_5 = \lambda_6 = 1$. They have the following geometrical meaning:

- The first pair (λ_1, λ_2) verify $\lambda_1 \cdot \lambda_2 = 1$, and are related to the hyperbolic character of the orbit. The related eigenvectors $e_1(0)$ and $e_2(0)$ give the most expanding and contracting directions.

- The second couple (λ_3, λ_4) are complex conjugate eigenvalues of modulus 1. The monodromy matrix, restricted to the plane spanned by the real and imaginary parts of the eigenvectors associated to λ_3, λ_4 is a rotation of angle Γ (the argument of λ_3).

- The third couple $(\lambda_5, \lambda_6) = (1, 1)$, is associated to the neutral directions (i.e. non-unstable modes). One eigendirection is the tangent vector to the orbit $(e_5(0))$. The other eigenvalue is associated to variations of the energy or any other variable which parametrises the family of periodic orbits.

Linear dynamics around a Periodic Orbit (II)

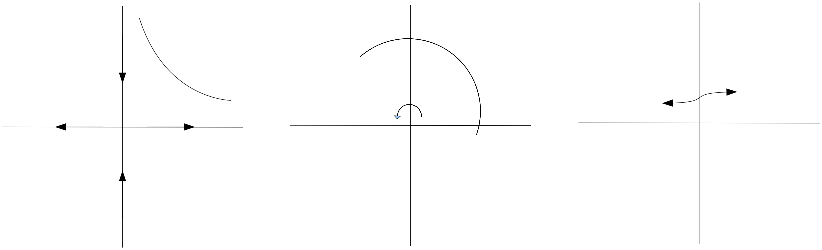
In a suitable basis, the monodromy matrix associated to a periodic orbit can be written in the form,

$$J = \left(\begin{array}{cc|cc|cc} \lambda_1 & & & & & \\ & \lambda_2 & & & & \\ \hline & & \cos\Gamma & -\sin\Gamma & & \\ & & \sin\Gamma & \cos\Gamma & & \\ \hline & & & & 1 & \varepsilon \\ & & & & 0 & 1 \end{array} \right).$$

The functions $e_i(\tau) = D\phi_\tau \cdot e_i(0)$, $i = 1, \ldots, 6$, give us an idea of the variation of the phase space properties in a small neighbourhood of the periodic orbit.

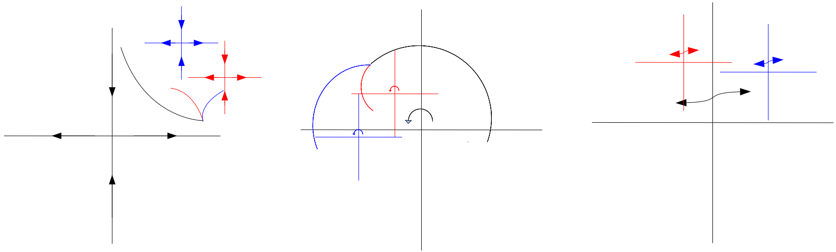
Linear dynamics around a Periodic Orbit (III)

The linear dynamics around these periodic orbit is a cross product between a saddle a centre and a neutral direction.



Linear dynamics around a Periodic Orbit (III)

The linear dynamics around these periodic orbit is a cross product between a saddle a centre and a neutral direction.



Appropriate changes on the sail orientation can make the trajectory come close to the Nominal Orbit.

As before, both the saddle projection and the centre directions must be taken into account when choosing a new sail orientation.

The Floquet Modes

Following [1] we define the Floquet modes $\bar{e}_i(\tau)$, $i = 1, \dots, 6$:

$$\left\{ \begin{array}{rcl} \bar{e}_1(\tau) & = & e_1(\tau) \exp\left(-\frac{\tau}{T} \ln \lambda_1\right), \\ \bar{e}_2(\tau) & = & e_2(\tau) \exp\left(-\frac{\tau}{T} \ln \lambda_2\right), \\ & & \\ \bar{e}_3(\tau) & = & \cos\left(-\frac{\Gamma\tau}{T}\right) e_3(\tau) - \sin\left(-\frac{\Gamma\tau}{T}\right) e_4(\tau), \\ \bar{e}_4(\tau) & = & \sin\left(-\frac{\Gamma\tau}{T}\right) e_3(\tau) + \cos\left(-\frac{\Gamma\tau}{T}\right) e_4(\tau), \\ & & \\ \bar{e}_5(\tau) & = & e_5(\tau), \\ e_6(\tau) & = & \bar{e}_6(\tau) + \epsilon(\tau)\bar{e}_5(\tau). \end{array} \right.$$

They $\bar{e}_i(\tau)$ are periodic functions and can easily be stored using a Fourier Series. We use the as a reference system around the orbit to track the relative position of the sail with a the reference orbit and the invariant manifolds.

---

[1] G. Gómez et al. *"Dynamics and Mission Design Near Libration Points - Volume I: Fundamentals: The Case of Collinear Libration Points"*, Volume 2 of World Scientific Monograph Series in Mathematics, 2001.

Schematic idea of the Station Keeping Algorithm

We look at the sails trajectory in the reference system

$$\{N_0(t); \bar{e}_1(t), \bar{e}_2(t), \bar{e}_3(t), \bar{e}_4(t), \bar{e}_5(t), \bar{e}_6(t)\}.$$

Hence,

$$z(t) = N_0(t^*) + \sum_{i=1}^{6} s_i(t^*)\bar{e}_i(t^*).$$

The station keeping algorithm:

1. Set $\alpha = \alpha_0, \delta = \delta_0$.
   When $|s_1(t^*)| \geq \varepsilon_{max}$ $\Rightarrow$ choose new sail orientation: $\alpha = \alpha_1, \delta = \delta_1$.
2. Set $\alpha = \alpha_1, \delta = \delta_1$.
   When $|s_1(t)|$ small $\Rightarrow$ restore the sail orientation: $\alpha = \alpha_0, \delta = \delta_0$.
3. Go Back to 1.

Finding $\alpha_{new}, \delta_{new}$

For each mission we will define the parameters ε_{max}, Dt_{min} and Dt_{max} that can vary for each mission application, and we will find α_{new}, δ_{new} and dt $\in [Dt_{min}, Dt_{max}]$ so that the trajectory comes back close to the nominal orbit.

- We use:

$$\mathcal{F}(dt, h_a, h_d) = \phi_{dt}(t_0, x_0, \alpha_0, \delta_0) + \frac{\partial \phi}{\partial \alpha}(t_0, x_0, \alpha_0, \delta_0) \cdot h_a + \frac{\partial \phi}{\partial \delta}(t_0, x_0, \alpha_0, \delta_0) \cdot h_d,$$

  to find dt, h_a and h_d so that the trajectory at time $t_0 + dt$ is close to the nominal orbit.

- For each $dt \in [Dt_{min}, Dt_{max}]$ we will find h_a and h_d such that $s_1 = 0$ and (s_5, s_6) are minimum (i.e. we are close to stable direction and the neutral direction is small).

- From all the dt, h_a and h_d we chose the one such that the other centre projection (s_3, s_4) is minimised.

$$\alpha_{new} = \alpha_0 + h_a^*, \qquad \delta_{new} = \delta_0 + h_d^*$$

Mission Scenarios

In the RTBPS we consider a Halo orbits $\Delta Z = 800.000$km. Taking $\beta = 0.05$ ($a_0 \approx 0.3$ mm/s^2), $\alpha_0 = 0^\circ$ and $\delta_0 = 0^\circ$.

Mission Parameters:

- We have taken $\varepsilon_{max} = 5 \cdot 10^{-5}$AU and $\varepsilon_{max} = 10^{-5}$AU (the escape distance), $dt_{min} = 30$ days and $dt_{max} = 115$ days (the minimum and maximum time between manoeuvres).

- We have done Monte Carlo Simulations for 1000 random initial conditions applying the station keeping strategy for 20 orbital revolutions.

- We have also tested the robustness of these strategies against different sources of errors. We have considered errors on the position and velocity determination, as well as errors in the sail orientation.

Results for the Monte Carlo Simulation

Table: Statistics for the simulations of the station keeping around a Halo orbit with $\Delta Z = 800.000$km. Different magnitudes on the error for the sail orientation are considered: `Err 1` $\rightarrow \epsilon_{\alpha,\delta} = 0.001^\circ$, `Err 2` $\rightarrow \epsilon_{\alpha,\delta} = 0.01^\circ$.

Sim. type	% Succ.	ε_{max} (AU)	$\Delta\alpha$ (deg)	$\Delta\delta$ (deg)
No Err	100 %	10^{-5}	0.023 - 0.005	0.038 - 0.001
No Err	100 %	$5 \cdot 10^{-5}$	0.113 - 0.030	0.191 - 0.002
Err 1	100 %	10^{-5}	0.023 - 0.004	0.046 - 0.001
Err 1	100 %	$5 \cdot 10^{-5}$	0.113 - 0.030	0.193 - 0.002
Err 2	18.7 %	10^{-5}	0.092 - 0.001	0.164 - 0.001
Err $2^\dagger$	99.6 %	$5 \cdot 10^{-5}$	0.121 - 0.016	0.250 - 0.001

Results for a 800.000km Halo Orbit

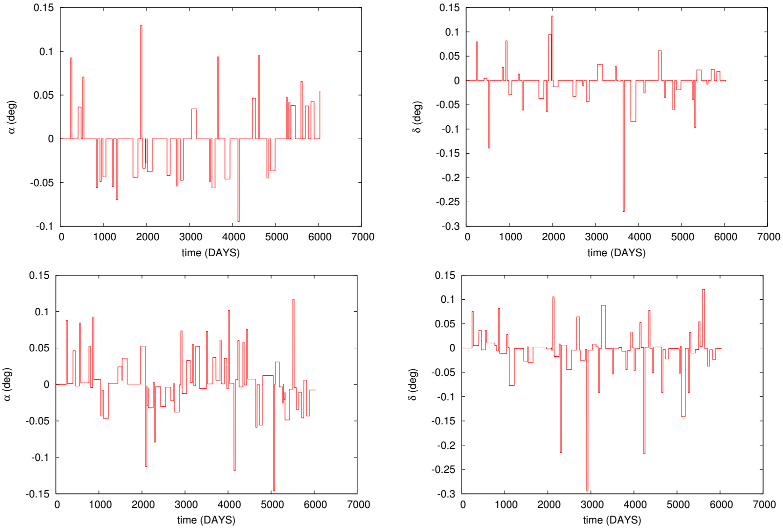
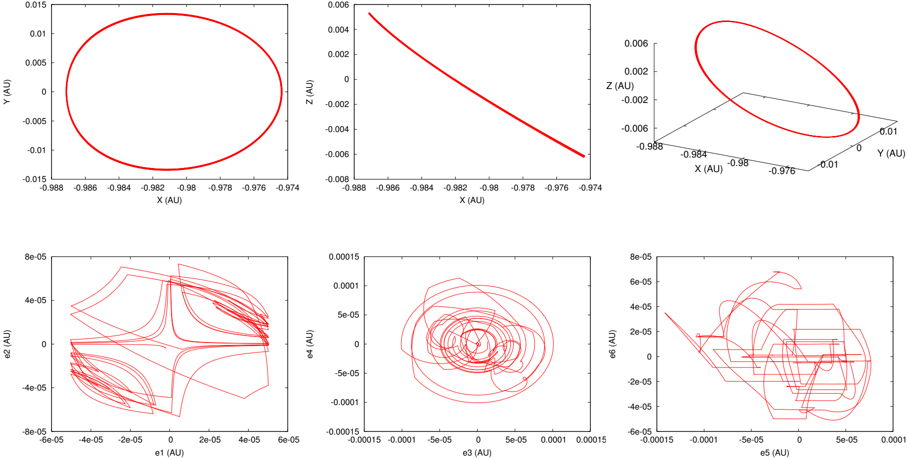


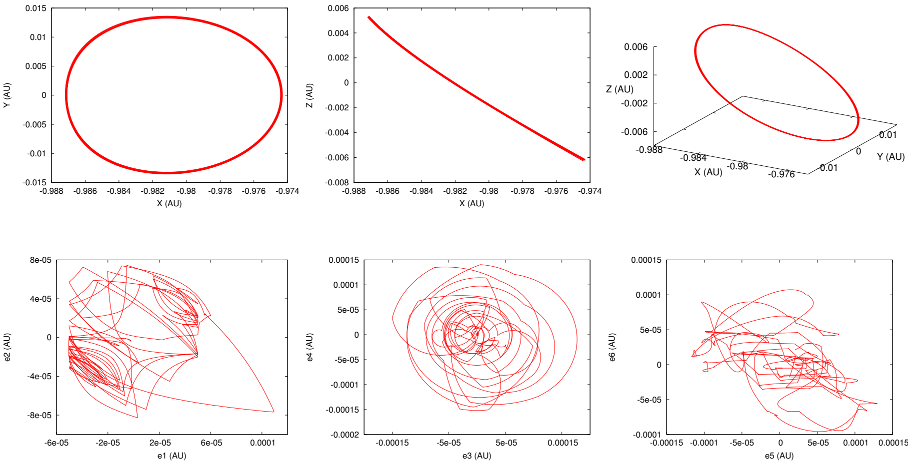
Figure: Vatiation of the sail orientation for a simulation with no errors (top) and with errors† (bottom) for a Halo orbit with $\Delta Z = 800.000$km.

Results for a 800.000km Halo Orbit



No Error on the simulations

Results for a 800.000km Halo Orbit



Error on the simulations

Mission Scenarios

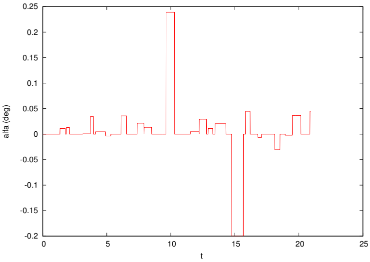
In the RTBPS we consider Vertical Lyapunov orbits around SL_2 with a period of 4 months. Here $\beta = 0.04$ ($a_0 \approx 0.24$ mm/s^2), $\alpha_0 = 0^\circ$ and $\delta_0 = 0^\circ$.

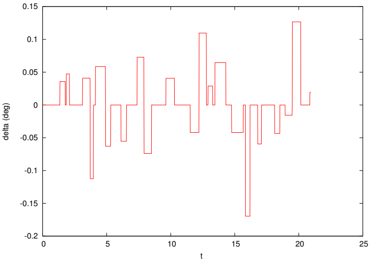
Mission Parameters:

- We have taken $\varepsilon_{max} = 5 \cdot 10^{-6}$AU (the escape distance), $dt_{min} = 20$ days and $dt_{max} = 60$ days (the minimum and maximum time between manoeuvres).
- We have applied the station keeping strategy for 10 orbital revolutions.

Results for a 4 month VLiap. Orbit

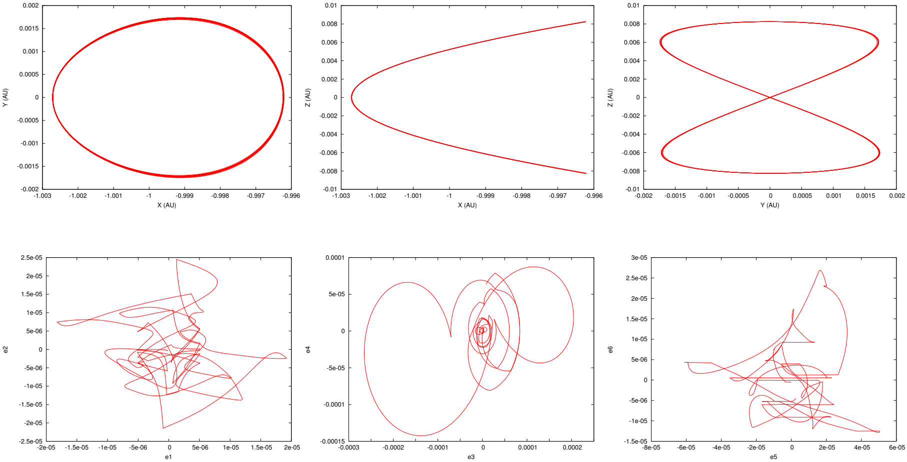
Variation of the sail orientation for one simulation.





Results for a 4 month VLiap. Orbit

Controled Trajectory for the solar sail.



Towards a More Realistic Model

Including more realism to the dynamical model

There are several ways to include more realism to the dynamical model. For example,

- taking a more realistic model for the Solar Sail by including the force produced by the absorption of the photons, the reflectivity properties of the sail material, ...
- taking a more realistic model for the gravitational perturbations by including the eccentricity in the Earth - Sun system. Or the gravitational attraction of other bodies, i.e. the Moon, Jupiter, ...

The result we will show here considered the eccentricity in the Earth - Sun system and the gravitational attraction of Jupiter. But similar results are obtained when we include the whole solar system.

The Dynamical Model

We use a Restricted N-Body Problem taking the Sun, Earth, Jupiter and solar sail, including the solar radiation pressure due to the solar sail.

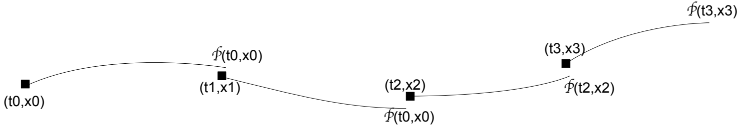
The equations of motion are,

$$\begin{aligned}
\ddot{x}_s &= \sum_{i=0}^{n} Gm_i \frac{x_i - x_s}{r_{is}^3} + \beta \frac{Gm_0}{r_{0s}^2} \langle \vec{r}_s, \vec{n} \rangle^2 n_x, & \ddot{x}_i &= \sum_{j \neq i} Gm_j \frac{x_i - x_j}{r_{ij}^3}, \\
\ddot{y}_s &= \sum_{i=0}^{n} Gm_i \frac{y_i - y_s}{r_{is}^3} + \beta \frac{Gm_0}{r_{0s}^2} \langle \vec{r}_s, \vec{n} \rangle^2 n_y, & \ddot{y}_i &= \sum_{j \neq i} Gm_j \frac{y_i - y_j}{r_{ij}^3}, \\
\ddot{z}_s &= \sum_{i=0}^{n} Gm_i \frac{z_i - z_s}{r_{is}^3} + \beta \frac{Gm_0}{r_{0s}^2} \langle \vec{r}_s, \vec{n} \rangle^2 n_z, & \ddot{z}_i &= \sum_{j \neq i} Gm_j \frac{z_i - z_j}{r_{ij}^3},
\end{aligned}$$

where (x_s, y_s, z_s) and (x_i, y_i, z_i) are the position of the solar sail and the planets respectively, where $i = 0, ..., n$ stands for the Sun, and the other planets.

Nominal Orbit

To find a good nominal orbit we have implemented a parallel shooting method to get a natural trajectory in the Sun - Earth - Jupiter model close to the fixed point in the RTBP Sun - Earth model.



- The points x_i that belong to the nominal orbit must satisfy:

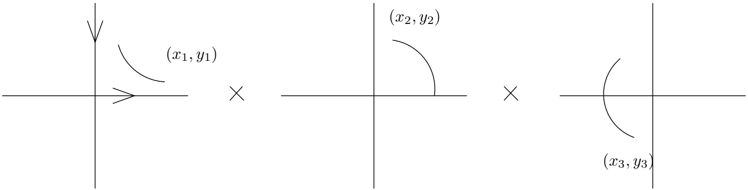
$$\phi_\tau(t_i, x_i) = x_{i+1} \quad \text{for } i = 0, \ldots, n-1.$$

- This leads to solving a non-linear equation with $6n$ equations and $6n+6$ unknowns.
- We have added six more conditions: we fix the initial positions (the first three components of x_0) and the final ones (the first three components of x_n).

Linear Dynamics

If we examine the eigenvalues of the variational flow along the nominal orbit we have: two real eigenvalues, $\lambda_1 > 0, \lambda_2 < 0$, and two pair of complex eigenvalues, $\nu_{1,2} \pm \mathrm{i}\omega_{1,2}$, with $|\nu_{1,2}| << |\lambda_{1,2}|$.

- A first good approximation for the linear dynamics along the nominal orbit is: saddle $\times$ centre $\times$ centre.



- To describe the trajectory of the sail along the orbit we will use the three reference planes defined by the corresponding eigenvectors.

Reference Frame

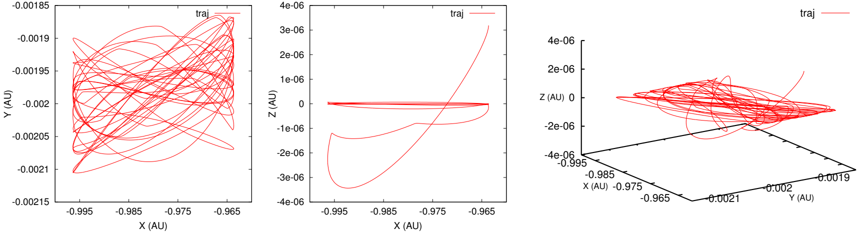
To avoid numerical problems with the computation of the eigenvectors, we split the nominal orbit into N revolutions (each revolution = 1 year). For each revolutions we can compute the Floquet modes and use them as a reference frame along the orbit.

$$
\begin{aligned}
\vec{v}_1(t) &= e_1(\tau)\exp\left(-\tfrac{\tau}{T}\ln\lambda_1\right), \\
\vec{v}_2(t) &= e_2(\tau)\exp\left(-\tfrac{\tau}{T}\ln\lambda_2\right), \\
\vec{v}_3(t) &= \left[\cos\left(-\Gamma_1\tfrac{\tau}{T}\right)e_3(\tau) - \sin\left(-\Gamma_1\tfrac{\tau}{T}\right)e_4(\tau)\right]\exp(-\tfrac{\tau}{T}\ln\Delta_1), \\
\vec{v}_4(t) &= \left[\sin\left(-\Gamma_1\tfrac{\tau}{T}\right)e_3(\tau) + \cos\left(-\Gamma_1\tfrac{\tau}{T}\right)e_4(\tau)\right]\exp(-\tfrac{\tau}{T}\ln\Delta_1), \\
\vec{v}_5(t) &= \left[\cos\left(-\Gamma_2\tfrac{\tau}{T}\right)e_5(\tau) - \sin\left(-\Gamma_2\tfrac{\tau}{T}\right)e_6(\tau)\right]\exp(-\tfrac{\tau}{T}\ln\Delta_2), \\
\vec{v}_6(t) &= \left[\sin\left(-\Gamma_2\tfrac{\tau}{T}\right)e_5(\tau) + \cos\left(-\Gamma_2\tfrac{\tau}{T}\right)e_6(\tau)\right]\exp(-\tfrac{\tau}{T}\ln\Delta_2),
\end{aligned}
$$

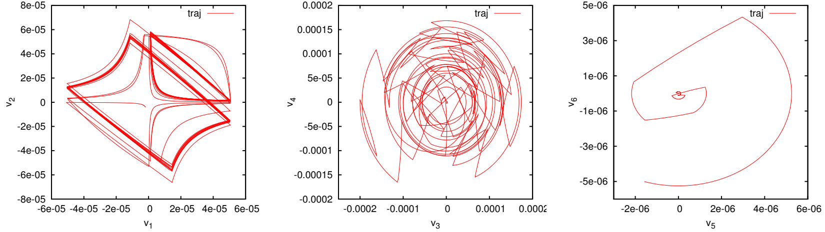
where $\Gamma_{1,2} = \arg(\lambda_{3,5})$.

Results for the Geostorm Mission

XY and XZ and XYZ Projections in a rotating reference system

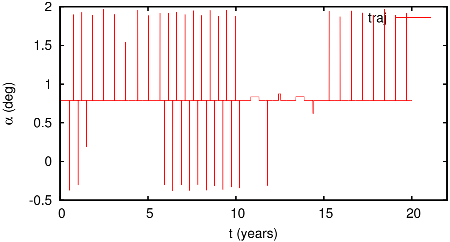


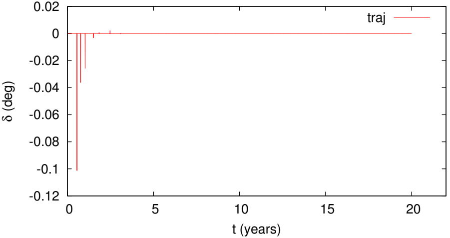
Saddle × Centre × Centre Projections



Results for the Geostorm Mission

Variation of the Sail Orientation





Conclusions & Future Work

Conclusion:

- We have derived station keeping strategies for a solar sail around an equilibrium point and periodic orbits.
- We have tested the robustness of these strategies when different sources of error occur during the simulations (position and velocity determination or solar sail orientation).
- We have extended these station keeping strategies to deal with a more more realistic model and applied them to the GeoStorm mission.
- Notice that these strategies do not require previous planning as the decisions are taken depending on the sails position at each time.

Future Work:

- Extend these ideas when the dynamics around a periodic orbit is Saddle x Saddle.
- Include a more realistic model for the performance of the sail.
- Compare these strategies with other control theory schemes such as LQR.

Thank you for your kind attention

☺ ☺ ☺ ☺ ☺