A numerical study of the Trojan dynamics

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Collaborations with:

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A. Jorba (Barcelona)
J. Laskar (Paris)

Restricted "several" bodies-problem

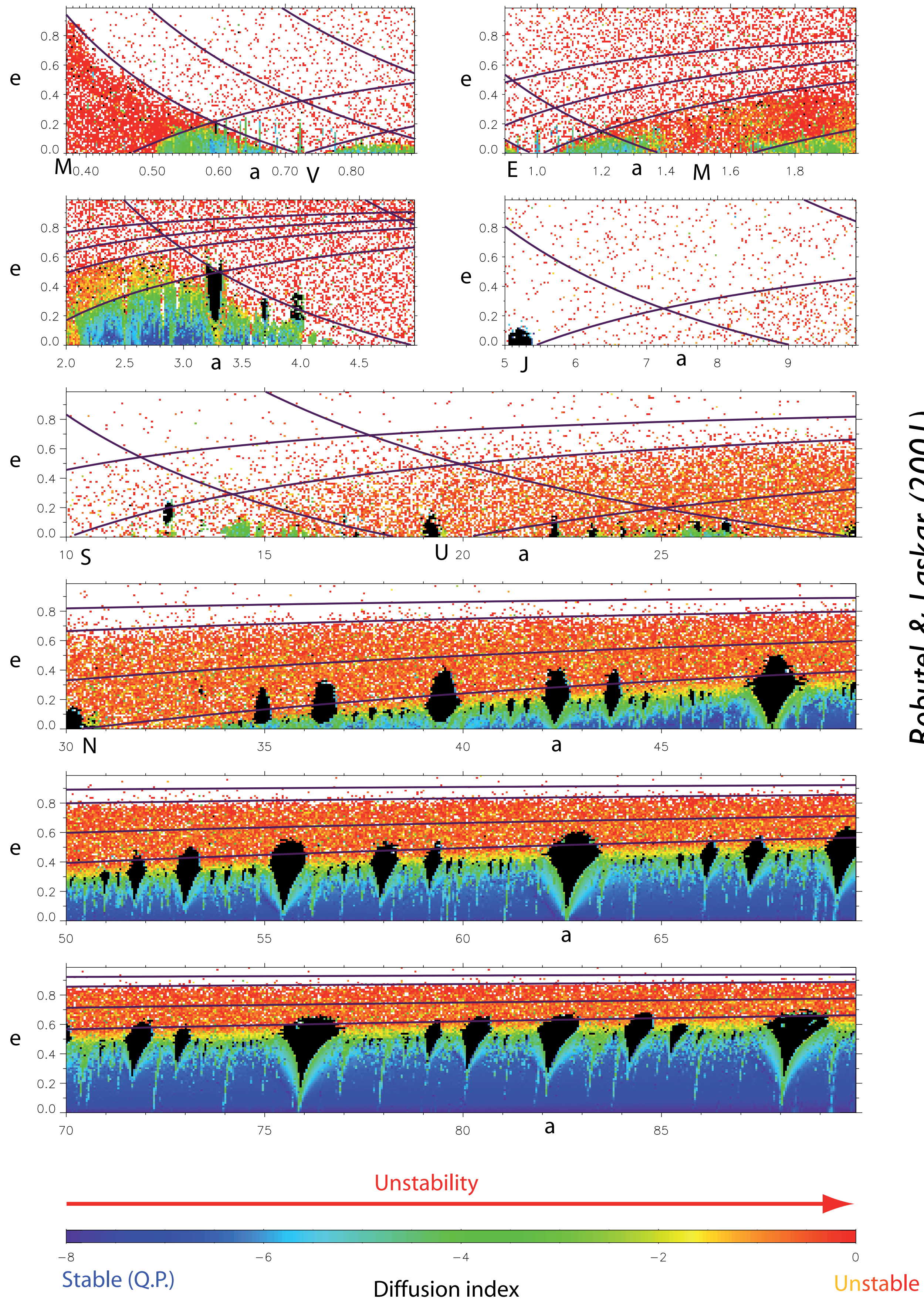
Different models:
- asteroid + Sun + 8 planets
- asteroid + Sun + 4 giant planets
- asteroid + Sun + Jup + Sat

From 3+8*3 = 24 to 3+2*3 = 9
d. f.

But even 9 D.F. imply numerical studies:
num. integrations of the trajectories
+
Analysis of the Traj.: Lypunov exponents,
Fourier analysis, Frequency Analysis...

Test-particles in the Solar system :

restricted 10 bodies-problem

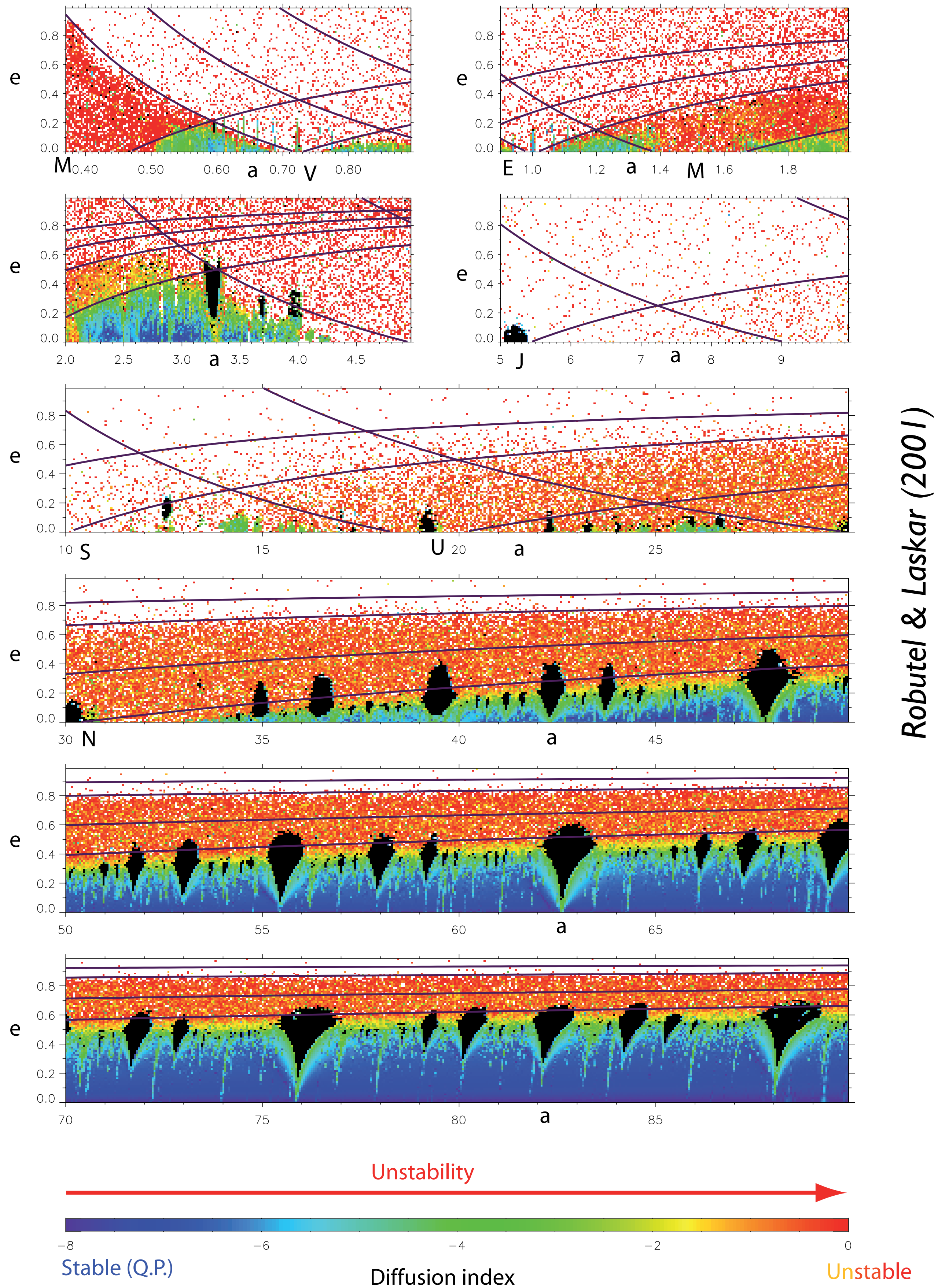


Robutel & Laskar (2001)

Test-particles in the Solar system :

restricted 10 bodies-problem

Sun & planets are given



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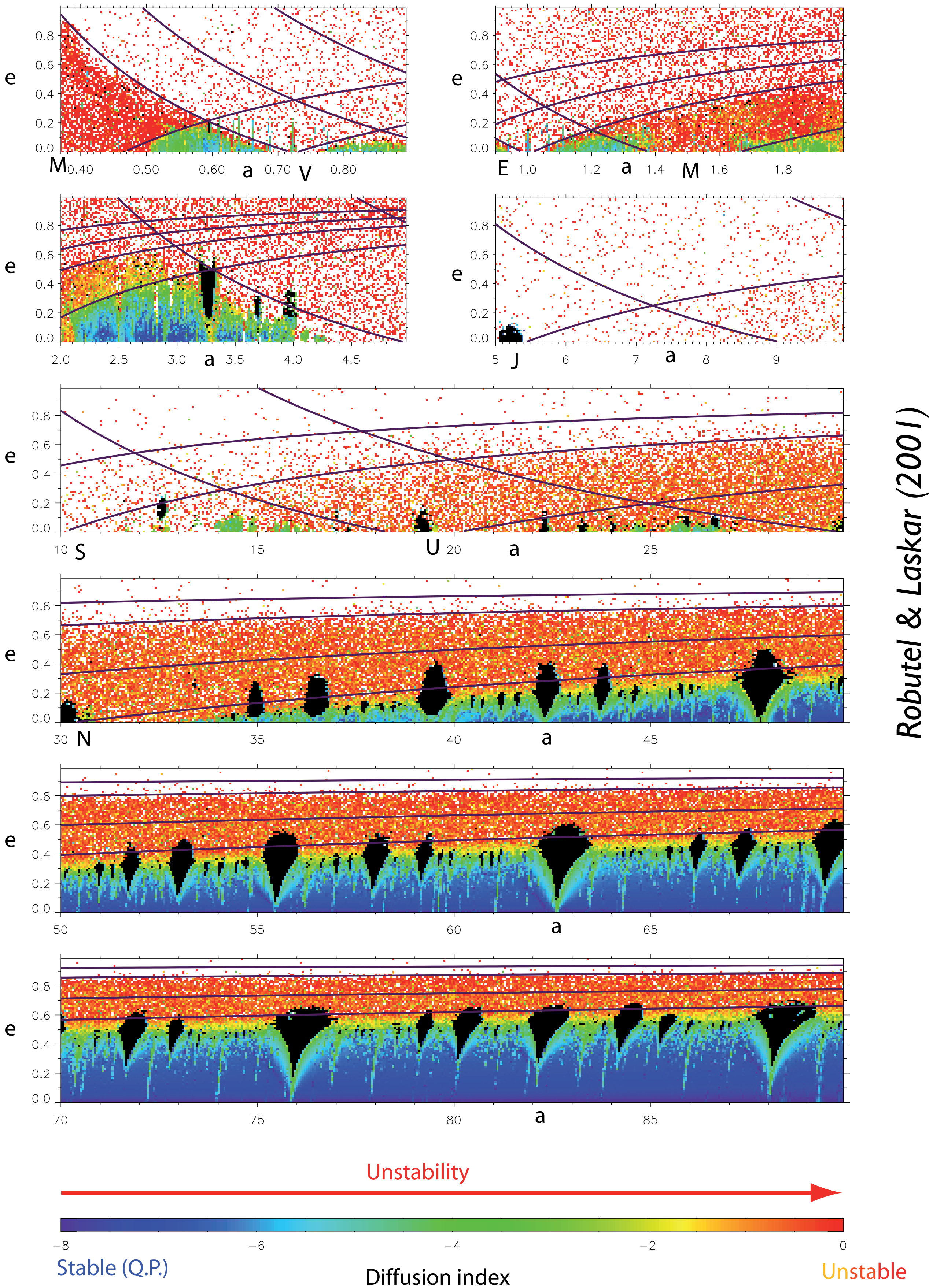

Sun & planets are given

$$a \in [0.38 : 90]\, A.U.$$

small body:

$$e \in [0 : 0.9]$$

$$I = 0$$

$$(\lambda, \varpi, \Omega)\ \text{Fixed}$$

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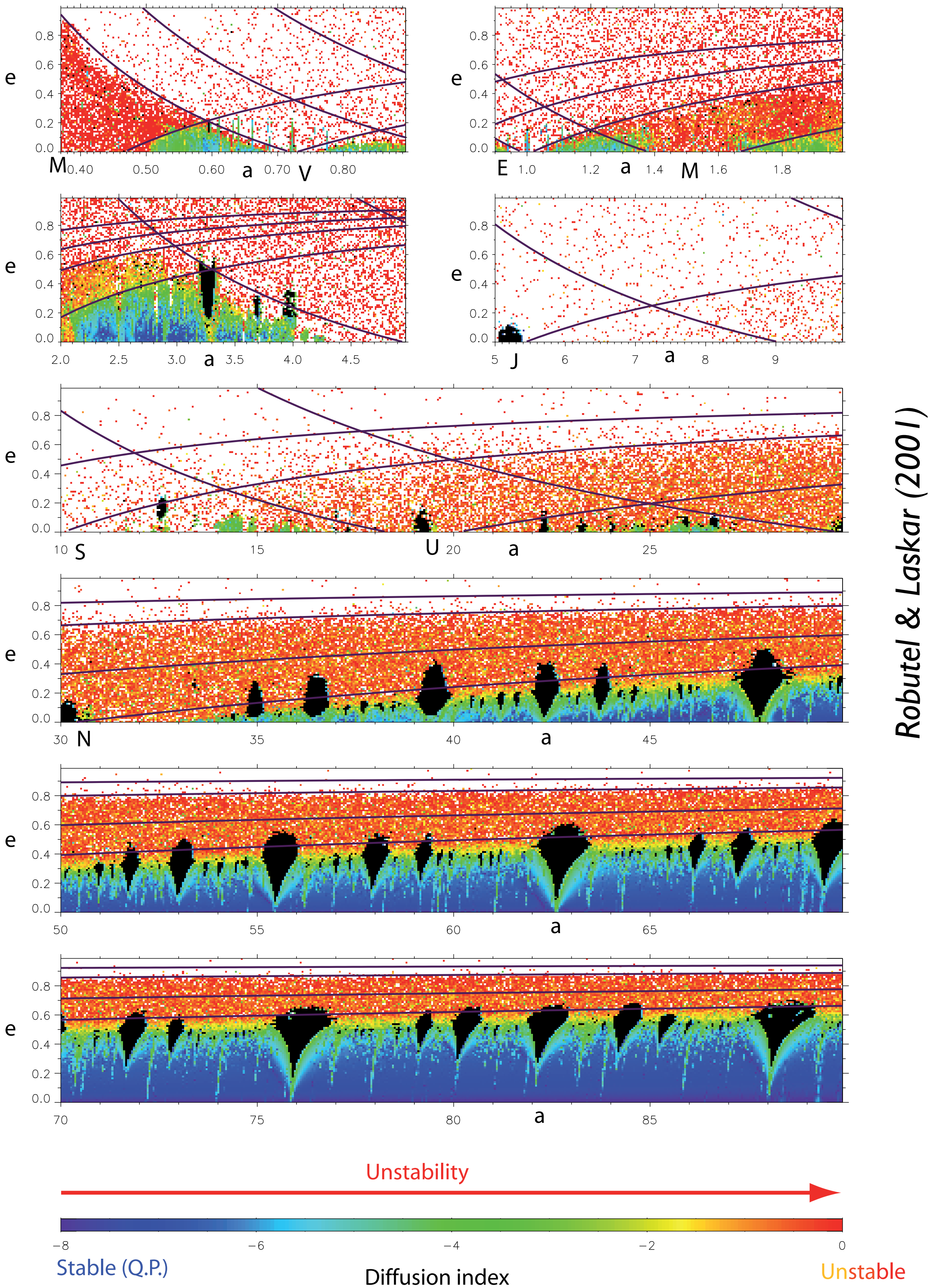

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Overlap of MMR above coll. lines:
Global chaos

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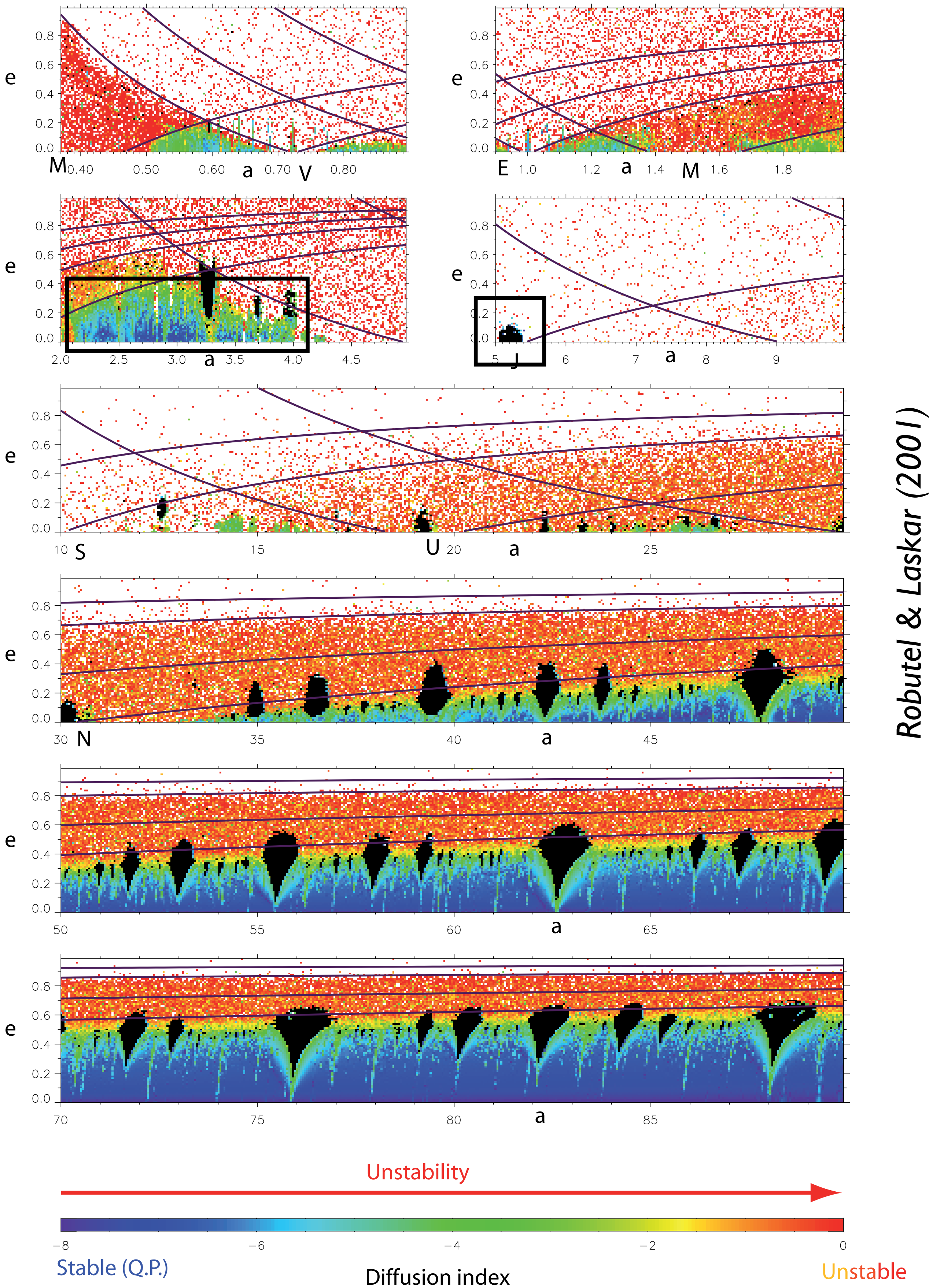

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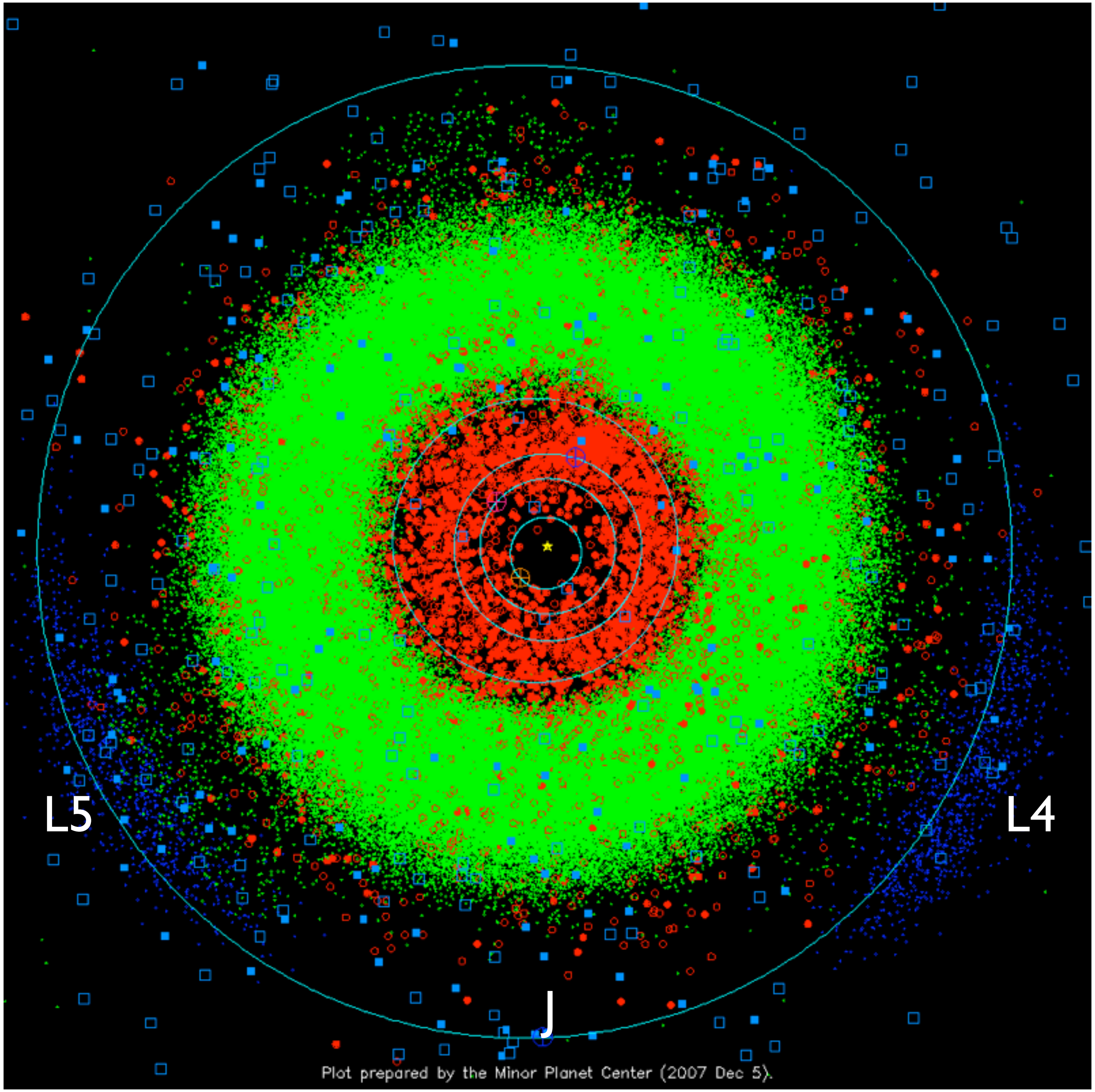
Projection of the observed inner solar system's objects on the ecliptic



Main asteroids belt
(~400000)

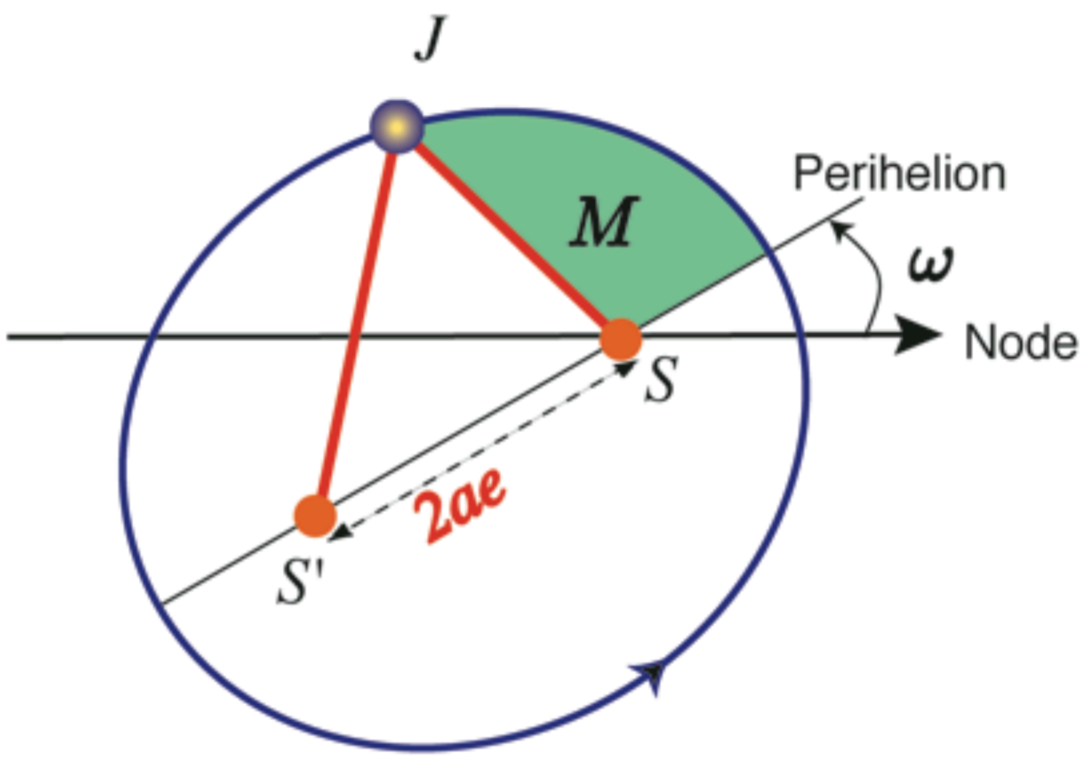
Terrestrial planet's crossers
(~5000)

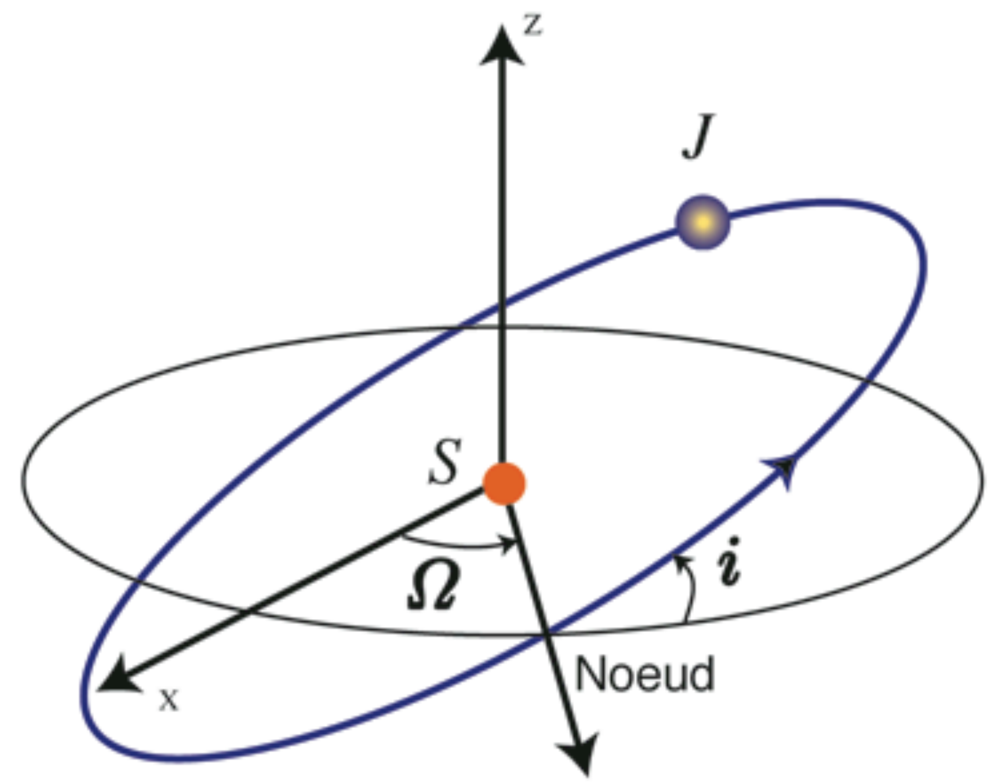
Comets
(~200)

Jupiter's trojans
(~2000)

Trojans can orbit far from
L4 or L5
(2 to 2.5 A.U.)

Kepler (1609)





ω : argument of the perihelion

e : eccentricity

Ω : longitude of the node

i : inclination

M : mean anomaly

a : semi-major axis

$\lambda = M + \varpi$: mean longitude

$\varpi = \omega + \Omega$: longitude of the perihelion

~ actions $(a, \; e, \; i,$

~ angles $M, \; \omega, \; \Omega)$

$M = nt$

fundamental Frequencies

(proper frequencies)

3 frequencies for the Trojan: (n, g, s)

Orbital motions: periods

3 months for Mercury

12 years for Jupiter

164 years for Neptune

1000 years at 100 A.U.

Secular motions: periods > 25000 years

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If Q.P. solution (evolves on a KAM torus)

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fundamental Frequencies
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If Q.P. solution (evolves on a KAM torus)

3n-1 planetary frequencies: (n_j, g_j, s_j) one of the $s_j = 0$

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The Frequency Map

J.Laskar (1999)

$$H(I,\theta) = H_0(I) + \varepsilon H_1(I,\theta) \qquad H \text{ real analytic for } (I,\theta) \in B^n \times \mathbb{T}^n$$

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If $\varepsilon = 0$

$$\begin{aligned} F : B^n &\longmapsto \Omega \subset \mathbb{R}^n \\ I &\longmapsto \nu(I) = \nabla H_0(I) \end{aligned}$$

$$\text{if } \; det\left(\frac{\partial^2 H_0(I)}{\partial I^2}\right) \neq 0$$

F is a diffeo. (loc.)

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If $\varepsilon \neq 0$ but small enough

There exists Ω_ε set of diophanine frequiencies $\leftrightarrow$ KAM tori

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Pöschel (1982),: There exists a diffeo. Ψ and a coord. syst. (φ, ν) such that

$$\Psi : \mathbb{T}^n \times \Omega \longmapsto \mathbb{T}^n \times B^n \qquad \Psi \text{ is analytical}/\varphi \text{ and } C^\infty \;/\nu$$

$$(\varphi, \nu) \longmapsto (\theta, I) \qquad \text{The flow is linear on: } \mathbb{T}^n \times \Omega_\varepsilon\text{:} \quad \dot{\nu} = 0 \,, \dot{\varphi} = \nu$$

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For fix $\theta \in \mathbb{T}^n$: $\quad \theta = \theta_0$

$$F_{\theta_0} : B^n \longrightarrow \Omega \;\; ; \qquad I \longrightarrow p_2(\Psi^{-1}(\theta_0, I))$$

The frequency map F_{θ_0} is a smooth diffeo. from, the actions space to the frequencies space

Goal

to obtain numerically a frequency map:

defined on B^n

which coincide with F_{θ_0} , up to numerical accuracy,
on the set of KAM tori

numerical tool: Frequency analysis (J.Laskar, 1988,1990)

Quasi-periodic decomposition of $ae^{i\lambda} = \sum_k \alpha_k e^{if_k}$

$\lvert\alpha_j\rvert$ (AU)	f_j rad/yr	combinations
46.183882	.02005033	n
.259757	.52968580	n_5
.058931	.21330868	n_6
.049411	.02004870	$n - g_8 + g$
.040885	.02005196	$n + g_8 - g$
.038045	.01808704	$-n + n_8 + g$
.031431	.02201360	$3n - n_8 - g$
$\vdots$	$\vdots$	

Quasi-periodic decomposition of $z_5 = e_5 \exp i\varpi_5$

$$z_5(t) \approx \sum_{j=1}^{N} \alpha_j \exp\left(i f_j t\right)$$

$$\begin{aligned} f_j = \quad & k_5 n_5 + k_6 n_6 \\ & p_5 g_5 + p_6 g_6 + q_6 s_6 \end{aligned}$$

Quasi-periodic decomposition of $z_5 = e_5 \exp i\varpi_5$

$\lvert\alpha_j\rvert$	f_j ("/yr)	k_5	k_6	p_5	p_6
4.41×10^{-2}	$+4.027603 \times 10^{0}$	+0	+0	+1	+0
1.59×10^{-2}	$+2.800657 \times 10^{1}$	+0	+0	+0	+1
6.44×10^{-4}	-2.126393×10^{4}	−1	+2	+0	+0
6.28×10^{-4}	$+5.198554 \times 10^{1}$	+0	+0	−1	+2
3.86×10^{-4}	$+1.411472 \times 10^{3}$	−2	+5	+0	−2
1.31×10^{-4}	$+2.270341 \times 10^{4}$	−1	+3	+0	−1
1.05×10^{-4}	-8.652321×10^{4}	−2	+3	+0	+0
9.92×10^{-5}	$+1.387493 \times 10^{3}$	−2	+5	+1	−3
8.06×10^{-5}	$+4.399535 \times 10^{4}$	+0	+1	+0	+0
6.45×10^{-5}	-4.255587×10^{4}	−2	+4	+0	−1
4.60×10^{-5}	-2.123995×10^{4}	−1	+2	−1	+1
4.28×10^{-5}	-2.128791×10^{4}	−1	+2	+1	−1
3.66×10^{-5}	-1.517825×10^{5}	−3	+4	+0	+0
3.49×10^{-5}	$+7.596451 \times 10^{1}$	+0	+0	−2	+3
3.45×10^{-5}	$+1.092546 \times 10^{5}$	+1	+0	+0	+0
2.54×10^{-5}	$+1.435452 \times 10^{3}$	−2	+5	−1	−1
2.01×10^{-5}	-1.078152×10^{5}	−3	+5	+0	−1
1.93×10^{-5}	-1.995139×10^{1}	+0	+0	+2	−1
1.85×10^{-5}	$+2.267943 \times 10^{4}$	−1	+3	+1	−2
1.82×10^{-5}	$+1.363514 \times 10^{3}$	−2	+5	+2	−4

$$z_5(t) \approx \sum_{j=1}^{N} \alpha_j \exp(i f_j t)$$

$$f_j = k_5 n_5 + k_6 n_6$$
$$p_5 g_5 + p_6 g_6 + q_6 s_6$$

Dynamical Maps and Frequency Analysis

C. I. planets $(a_j, e_j, I_j, \lambda_j, \varpi_j, \Omega_j)$ Fixed

(n_j, g_j, s_j) given

$(a^{(0)}, e^{(0)})$ $\xrightarrow{F_{I^{(0)}}}$ (n, g, s)

$(\lambda^{(0)}, \varpi^{(0)}, \Omega^{(0)})$

$I^{(0)}$ fixed

2-d surface in $\mathbb{R}^3$

$[0, T]$ $[T, 2T]$

Frequency Analysis
(Laskar)

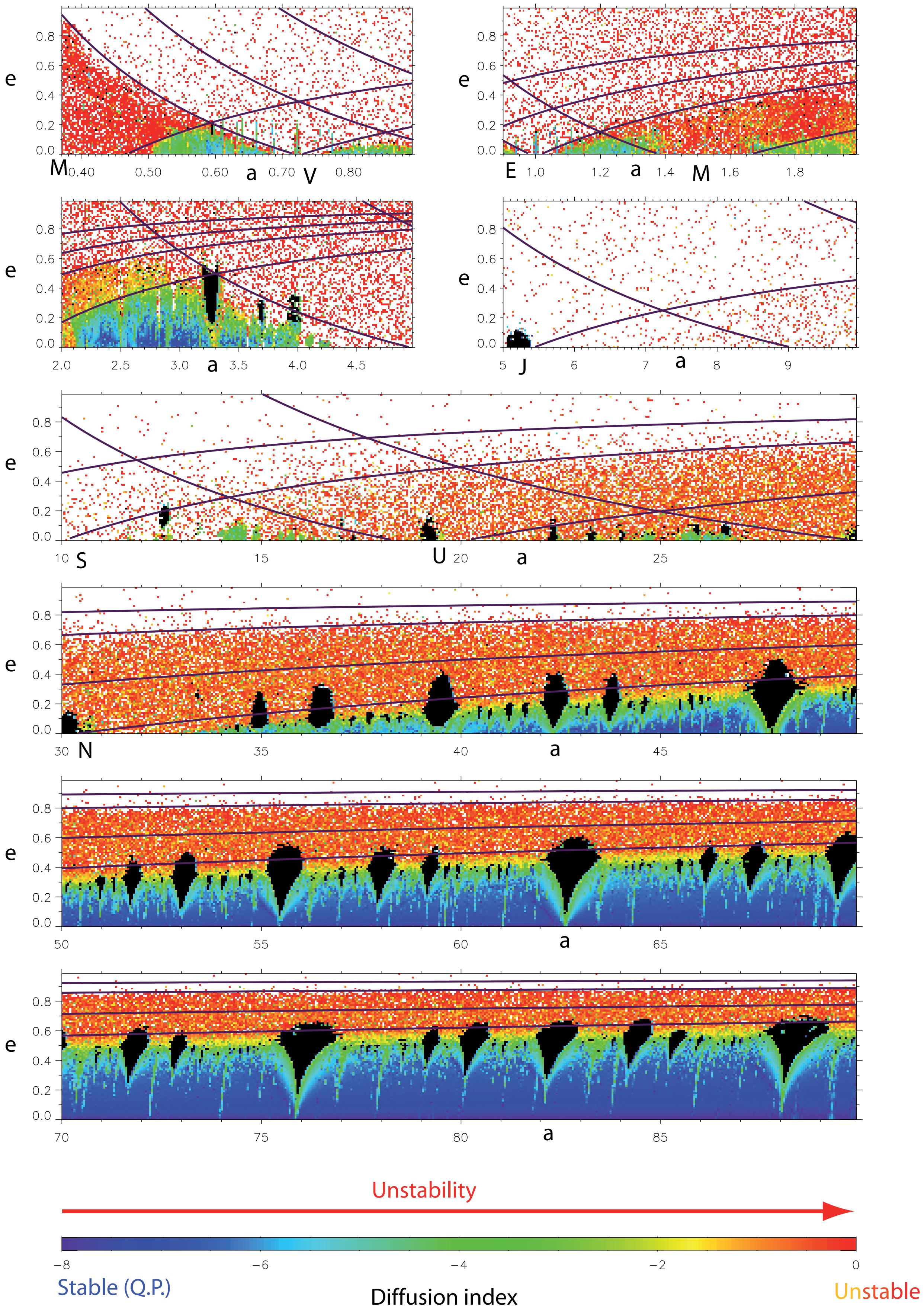
f_1 + f_2 → Diffusion

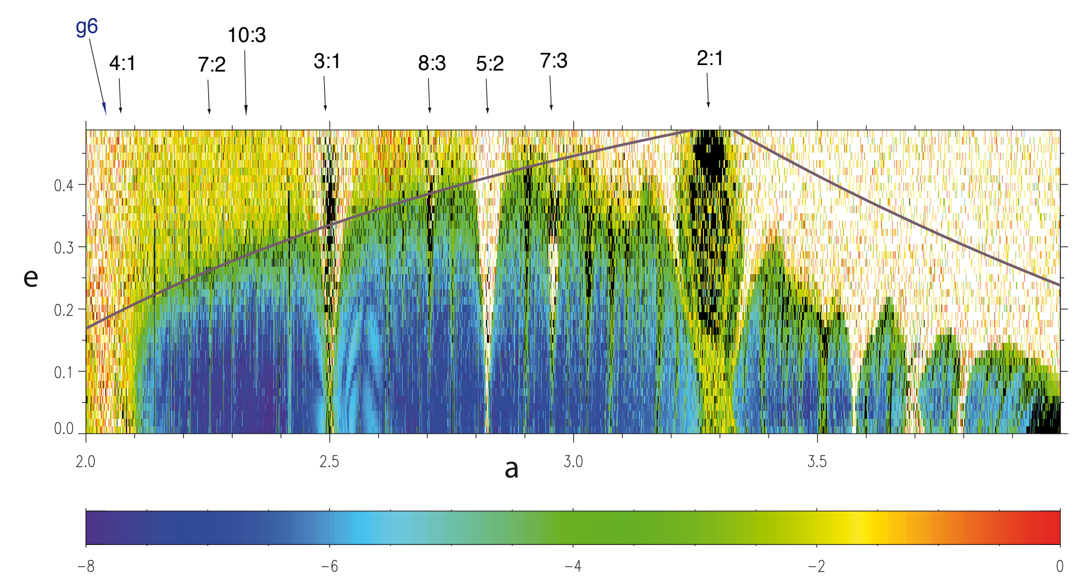
$$\sigma = \log_{10}\left(||f_1 - f_2|| / ||f_1||\right)$$

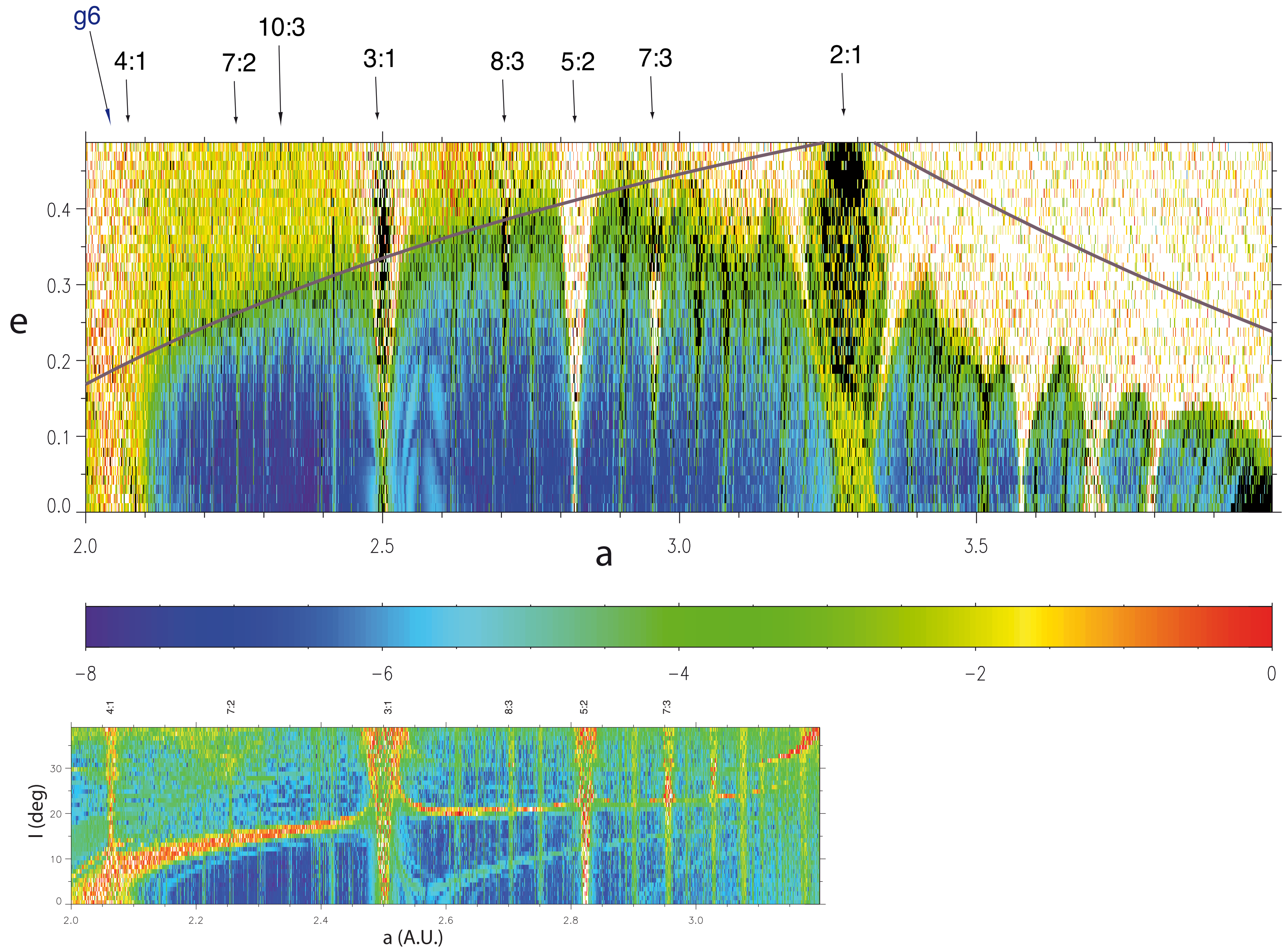
Dynamical map in the
"frequency space"

↔ correspondance

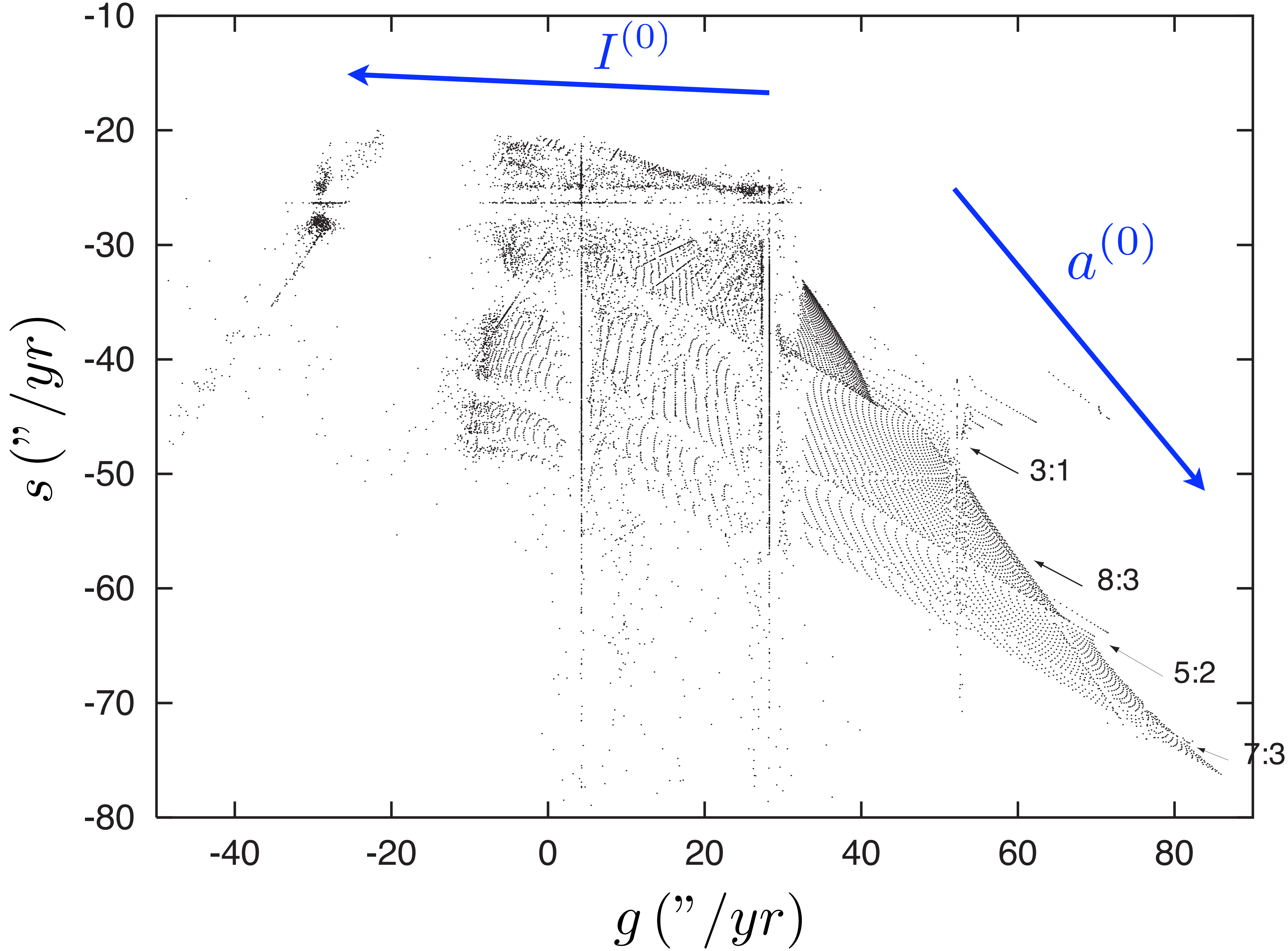
Dynamical map in the
"action space"
(a,e)


M
V
E
M
J
S
U
N
e
a
Unstability
Stable (Q.P.)
Diffusion index
Unstable

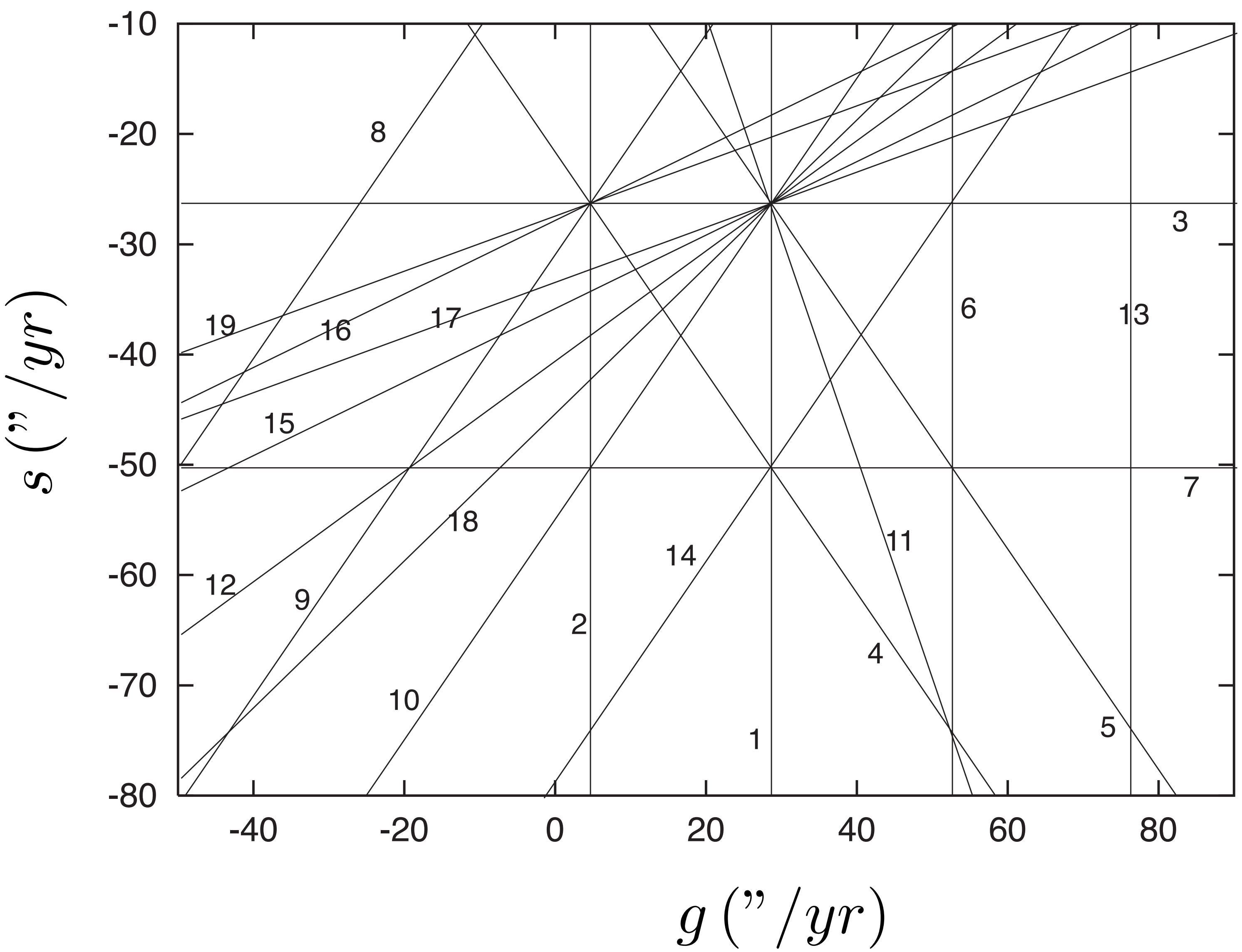

g6
4:1
7:2
10:3
3:1
8:3
5:2
7:3
2:1
e
a


g6
4:1
7:2
10:3
3:1
8:3
5:2
7:3
2:1
e
0.4
0.3
0.2
0.1
0.0
2.0
2.5
3.0
3.5
a
−8
−6
−4
−2
0
4:1
7:2
3:1
8:3
5:2
7:3
I (deg)
30
20
10
0
2.0
2.2
2.4
2.6
2.8
3.0
a (A.U.)

Frequency Map (secular)

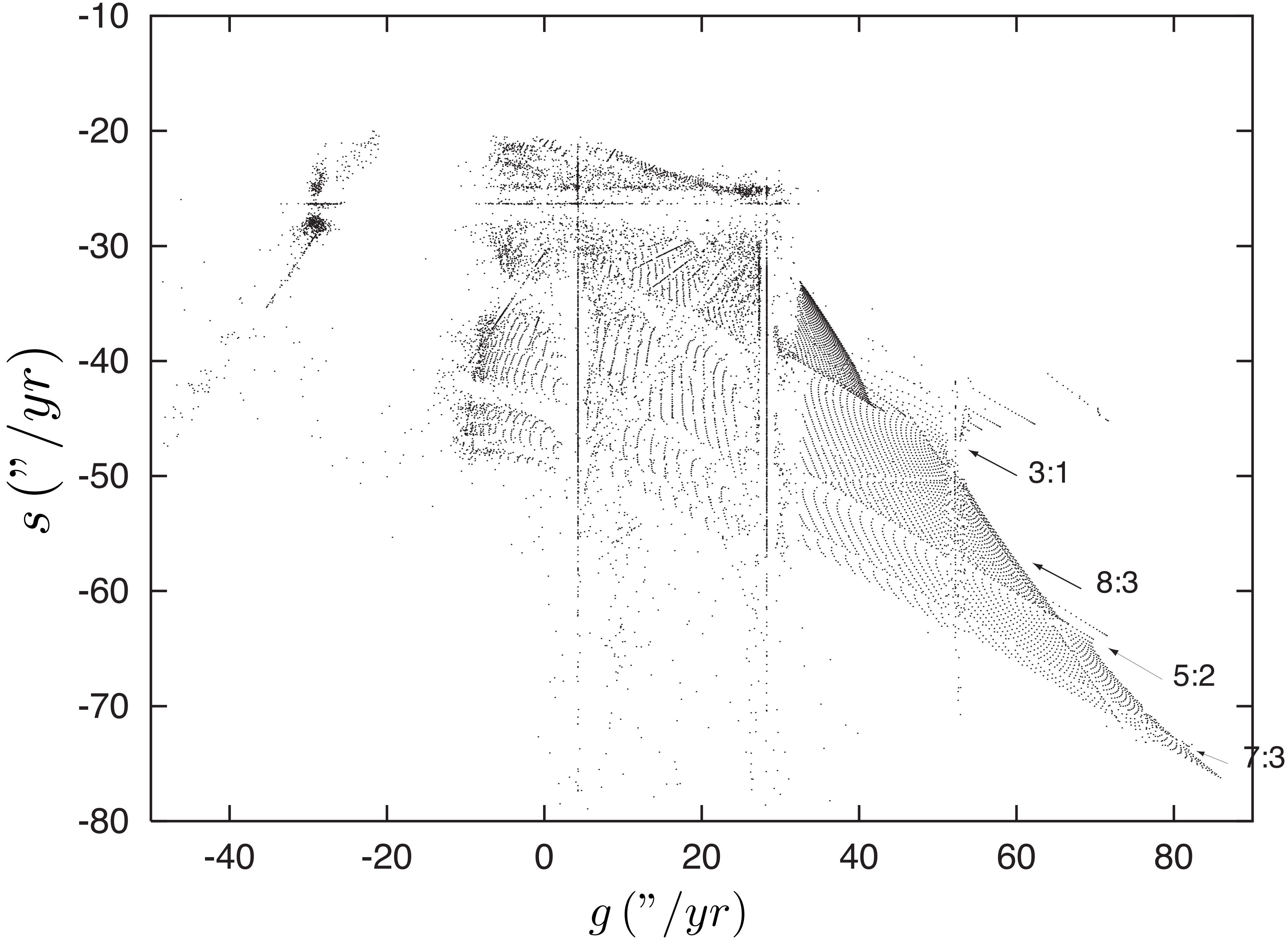


Main secular resonances in the asteroid belt

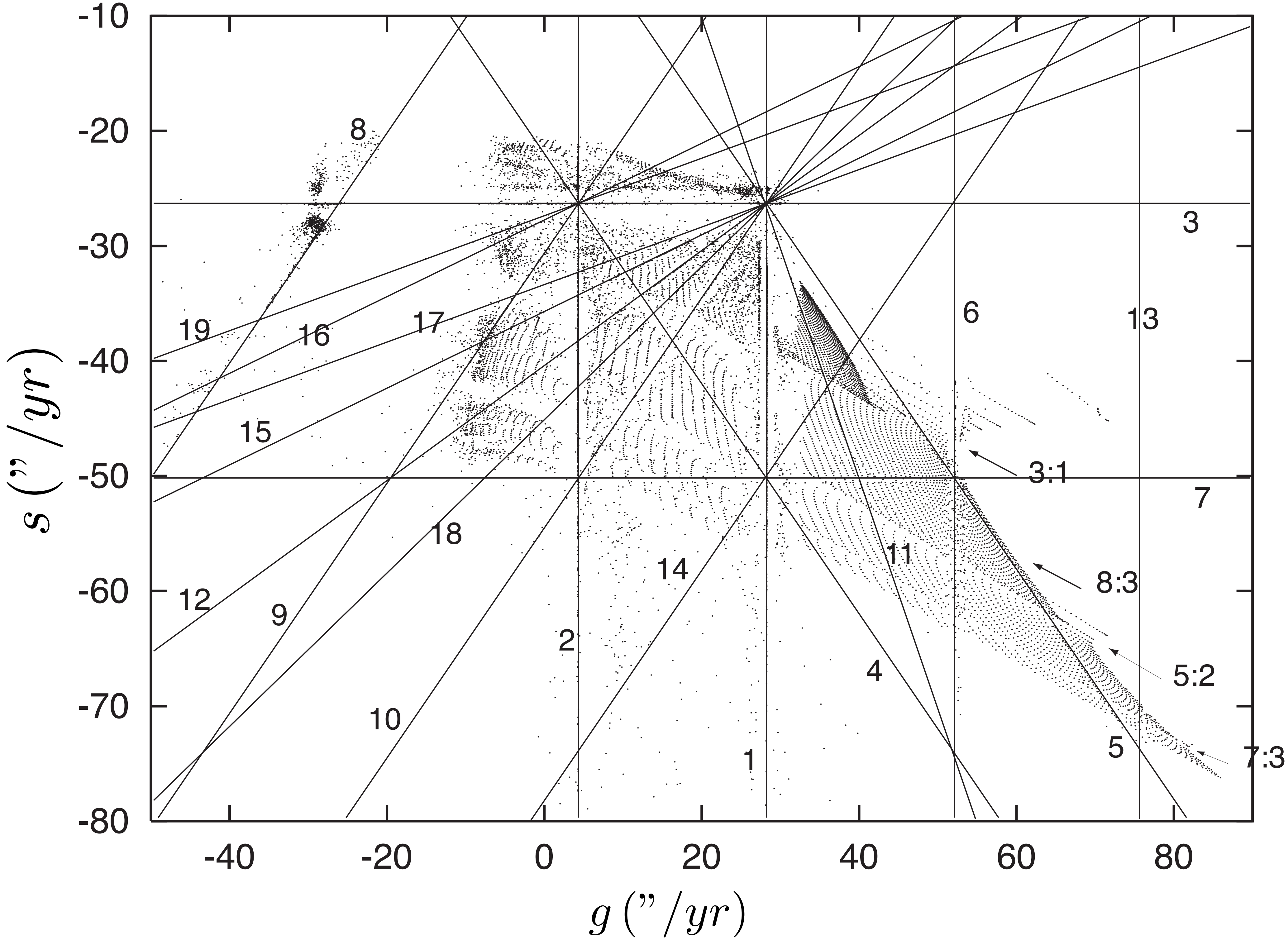


$$pg + qs + r_1 g_5 + r_2 g_6 + r_3 s_6 = 0$$

Label	p	q	r_1	r_2	r_3
1	1	0	0	-1	0
2	1	0	-1	0	0
3	0	1	0	0	-1
4	1	1	-1	0	-1
5	1	1	0	-1	-1
6	1	0	1	-2	0
7	0	1	-1	1	-1
8	2	-2	0	0	0
9	1	-1	-1	0	1
10	1	-1	0	-1	1
11	2	1	0	-2	1
12	1	-2	0	-1	2
13	1	0	2	-3	0
14	1	-1	1	-2	1
15	1	-3	0	-1	3
16	1	-3	-1	0	3
17	1	-4	0	-1	4
18	2	-3	0	-2	3
19	1	-4	-1	0	4

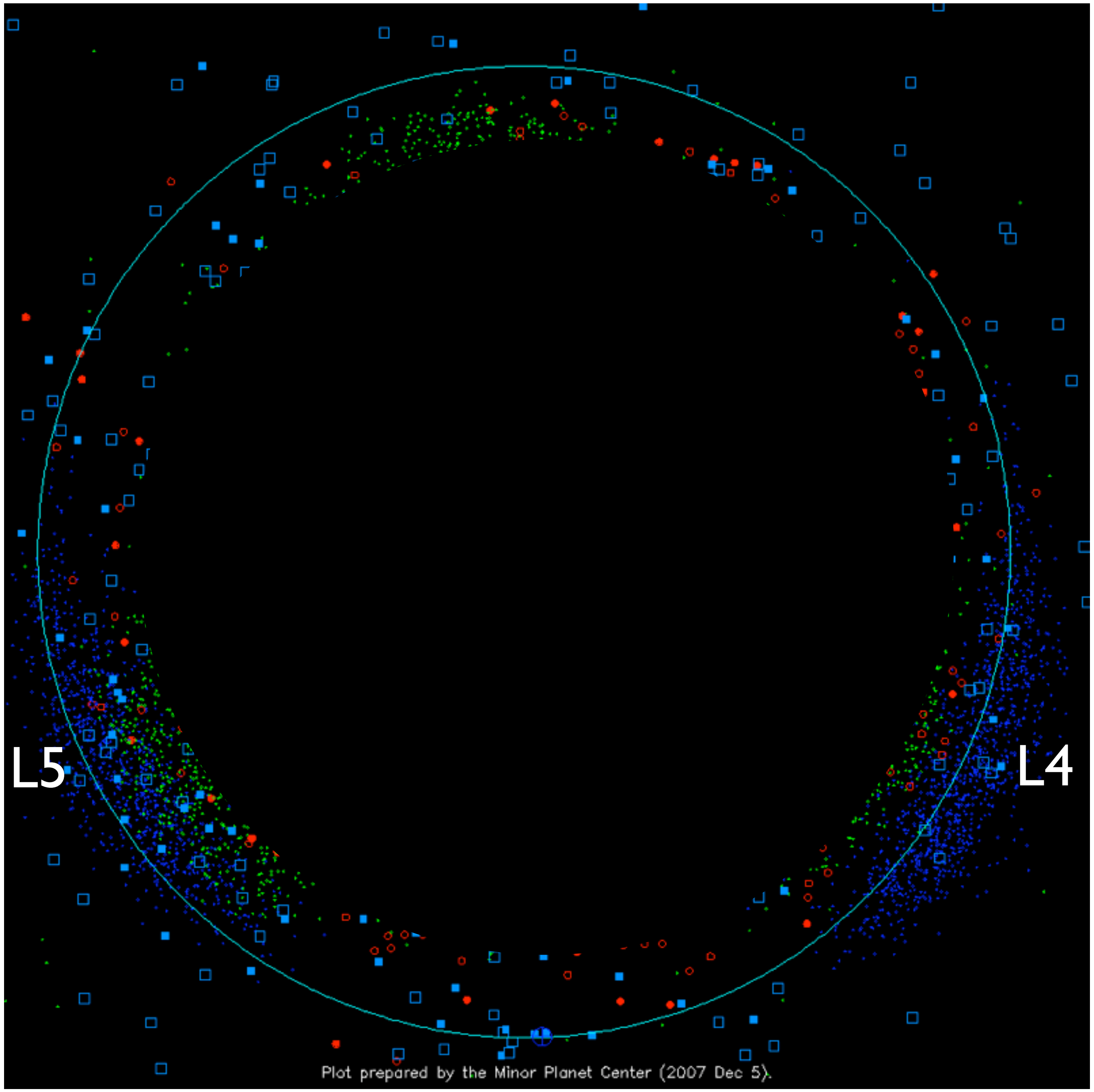


-10
-20
-30
-40
-50
-60
-70
-80
-40
-20
0
20
40
60
80
$s\,('' / yr)$
$g\,('' / yr)$
3:1
8:3
5:2
7:3



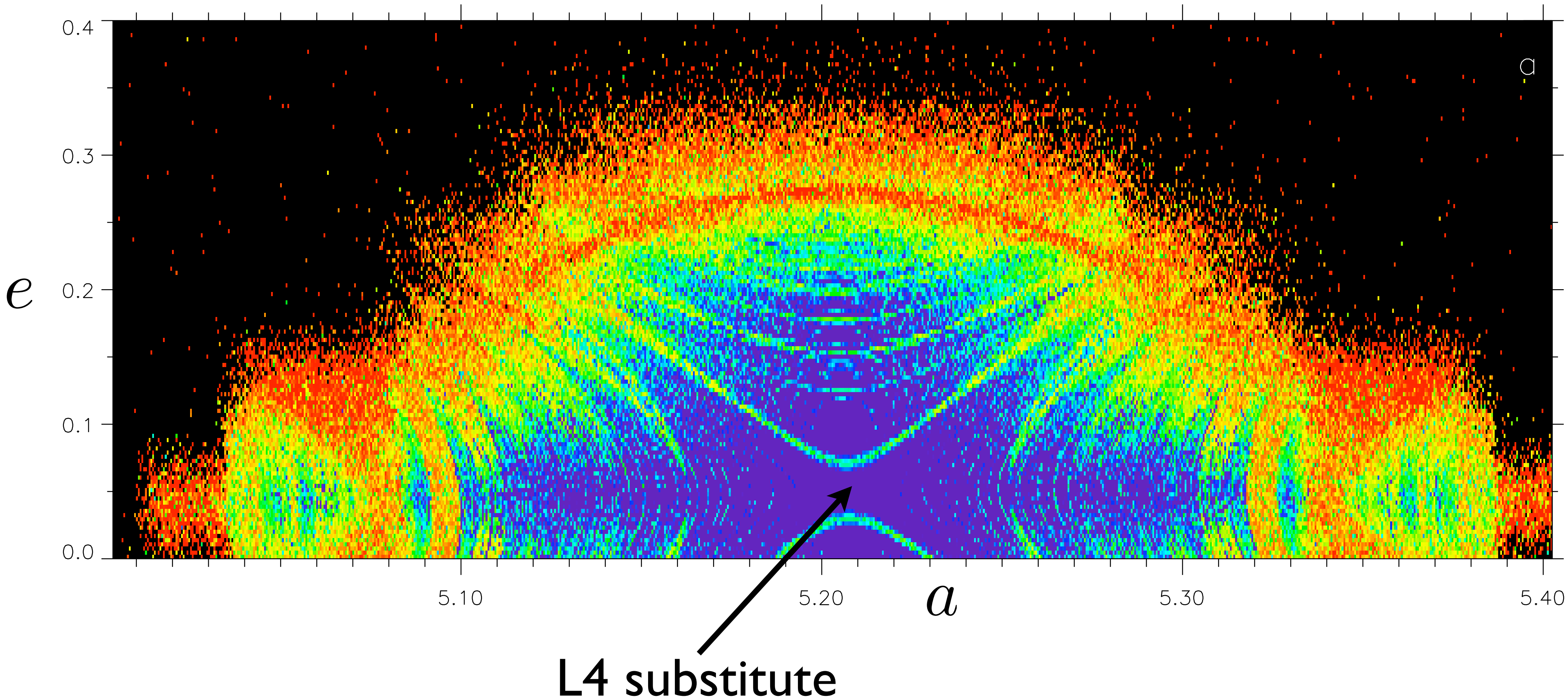
-10
-20
-30
-40
-50
-60
-70
-80
-40
-20
0
20
40
60
80
s (”/yr)
g (”/yr)
8
3
19
16
17
6
13
15
3:1
7
18
11
14
8:3
12
9
2
5:2
4
10
5
7:3
1

Jovian Trojans



Jupiter's trojans
(~2000)

Robutel, Gabern & Jorba (2005, 2006)



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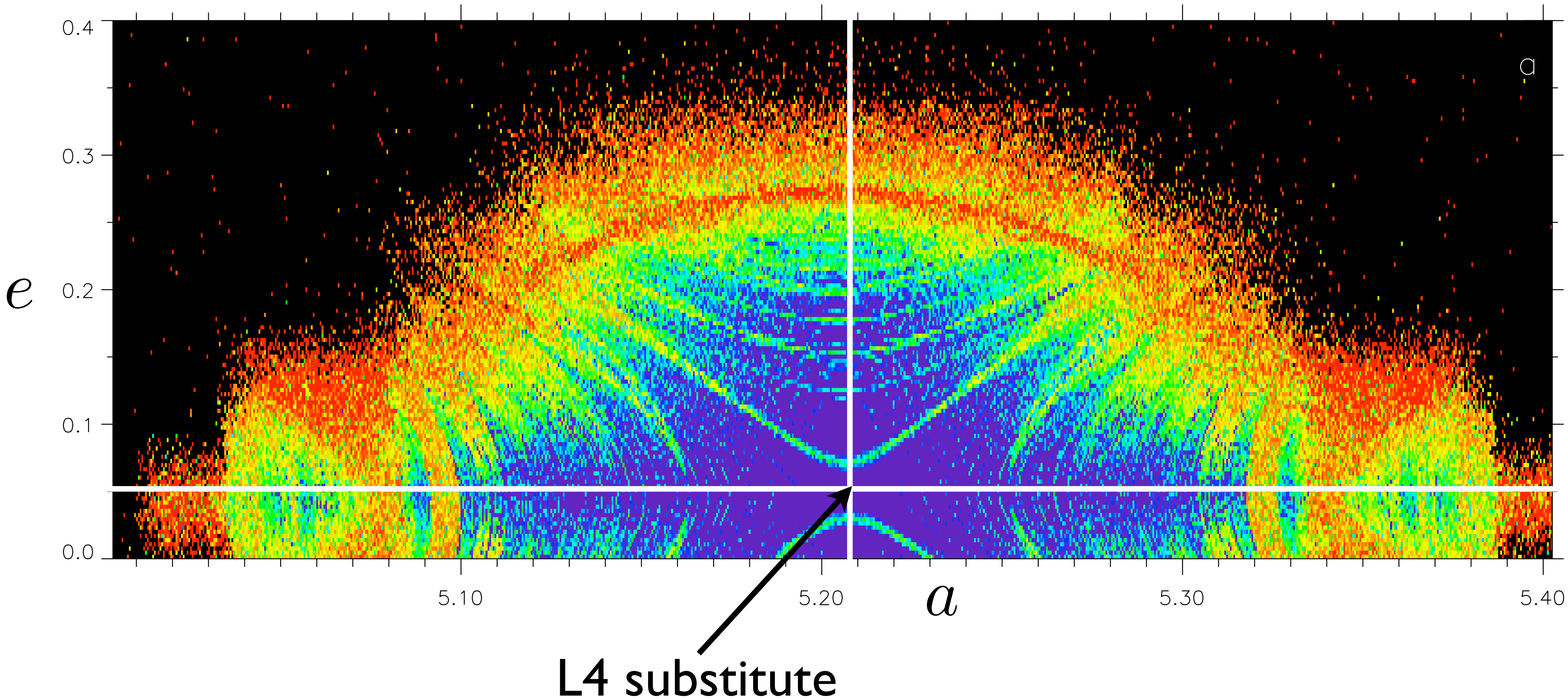
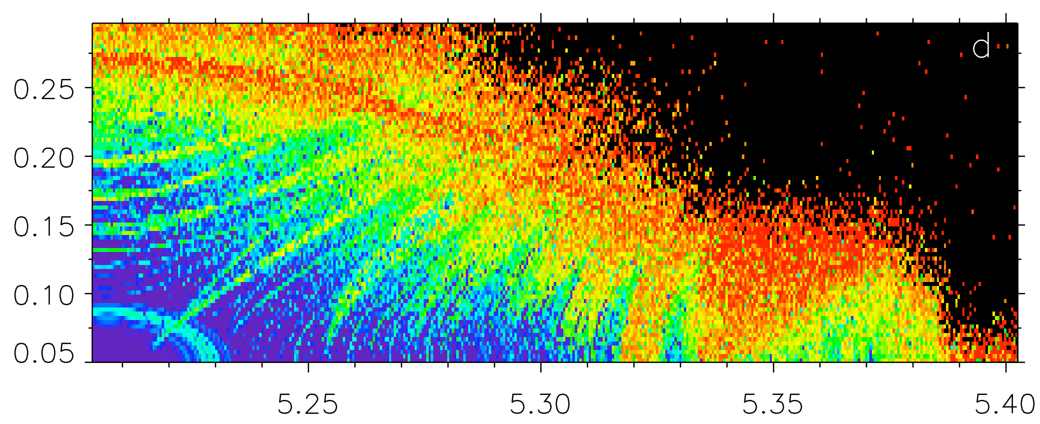


Comparison of different models



R.4.B.P. (Sun+J+Sat+T)

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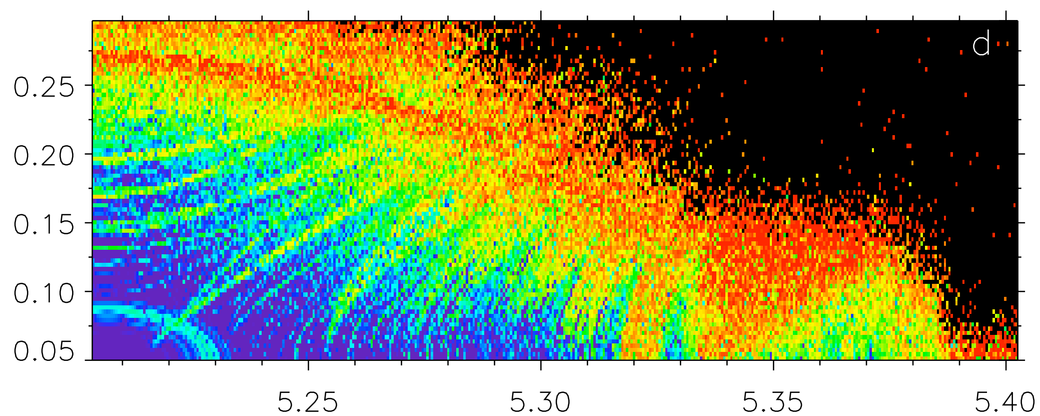

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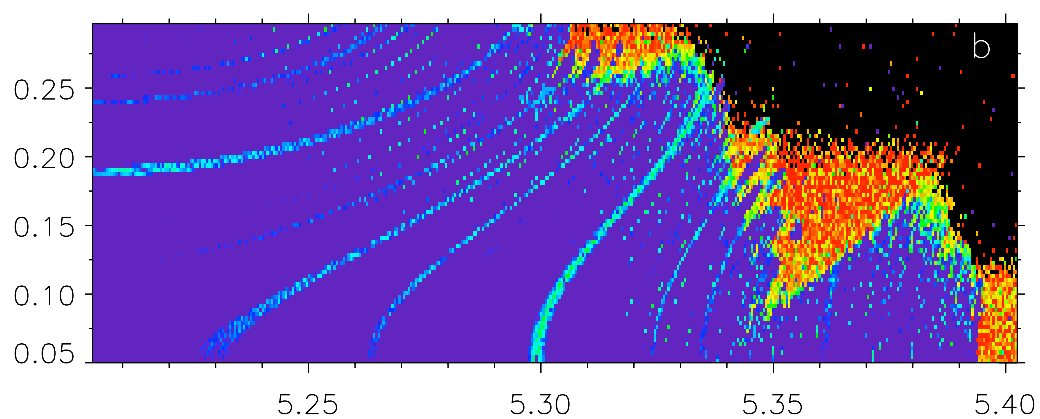

E.R.T.B.P. (Sun+J+T)

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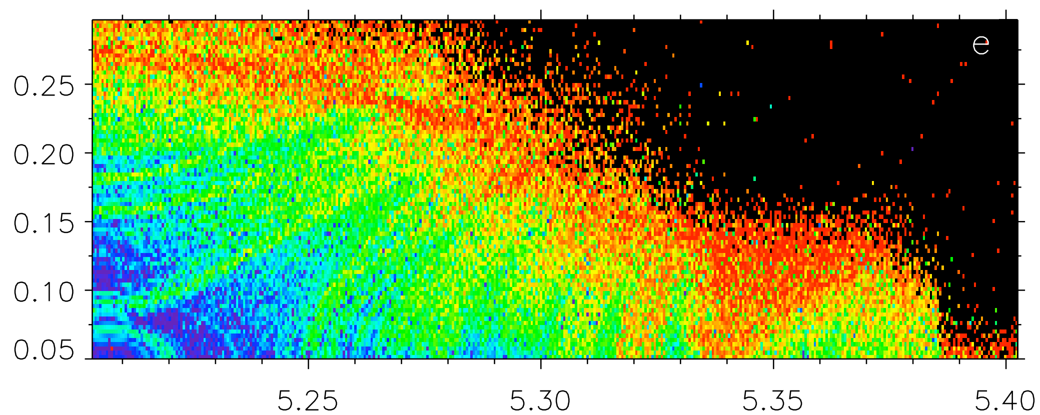
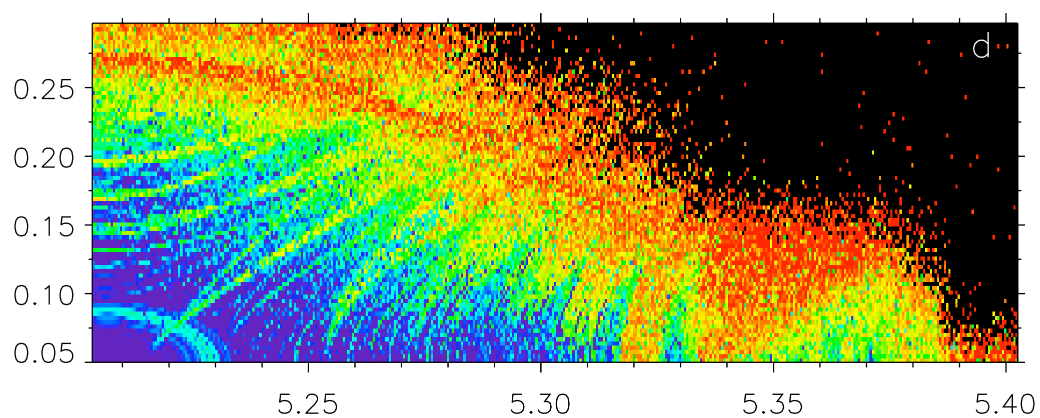
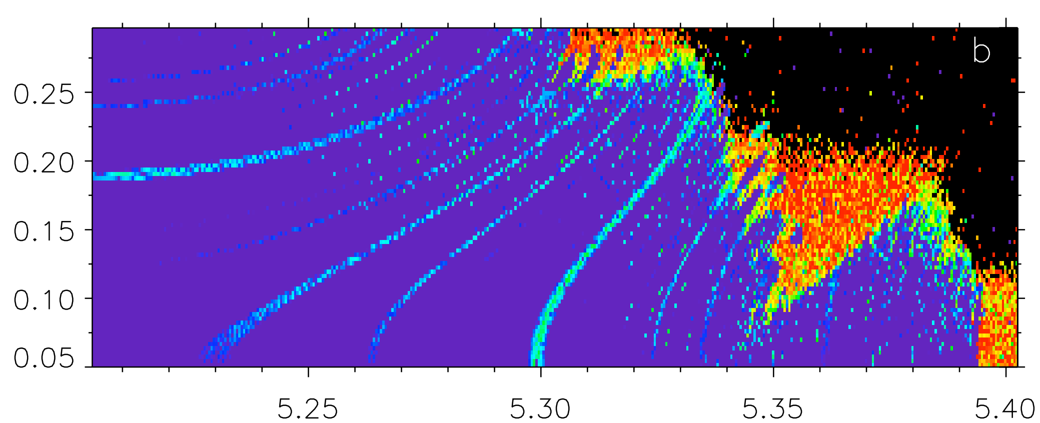


R.6.B.P. (Sun+J+Sat+U+N+T)



R.4.B.P. (Sun+J+Sat+T)



E.R.T.B.P. (Sun+J+T)

fundamental Frequencies

(proper frequencies)

5 planetary frequencies : $(n_5, n_6, g_5, g_6, s_6)$

3 for a test-particle : (n, g, s)

Trojan : 1:1 orbital resonance $n_5 = n$

(ν, g, s)

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$$(\nu, g, s)$$

$\nu \in [7500, 9200]$ arcsec/year $T_\nu \in [140, 155]$ years

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$g \in [230, 450]$ arcsec/year $T_g \in [2880, 5634]$ years

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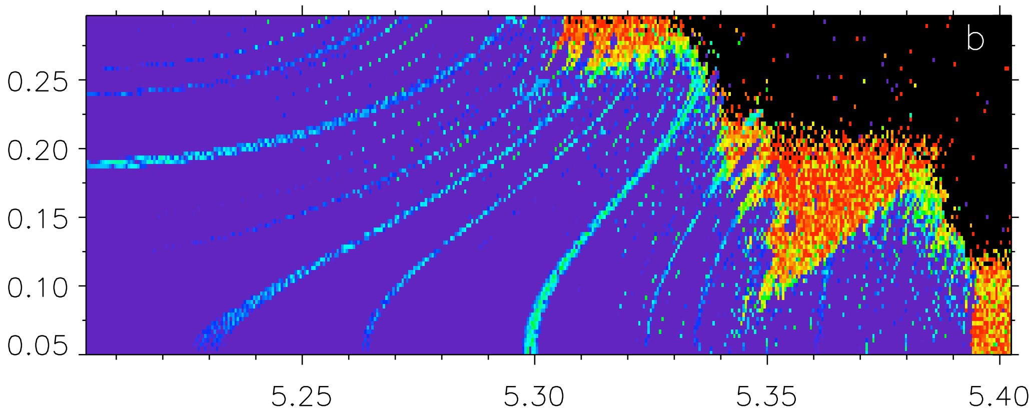
$g \in [230, 450]$ arcsec/year $T_g \in [2880, 5634]$ years

$s \in [-50, 5]$ arcsec/year $T_s > 25000$ years

2 obvious families of resonances

Family I $p\nu = n_5$



E.R.T.B.P. (Sun+J+T)

$$\mathrm{p}\nu - n_5 + qg = 0$$

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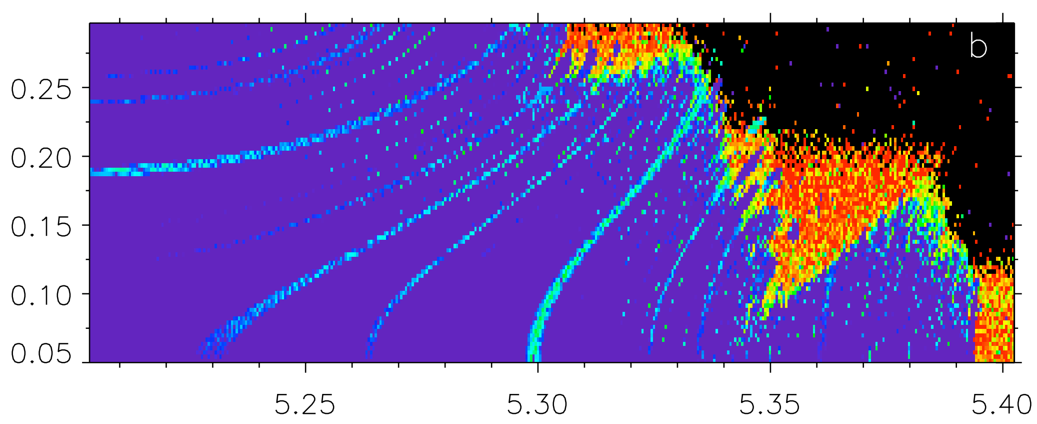

E.R.T.B.P. (Sun+J+T)

$$\mathrm{p}\nu - n_5 + qg = 0$$

Family III: Secular resonances

$$kg + ls + k_5 g_5 + k_6 g_6 + l_6 s_6 = 0$$

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6.44×10^{-4}	-2.126393×10^{4}	-1	$+2$	$+0$	$+0$
6.28×10^{-4}	$+5.198554 \times 10^{1}$	$+0$	$+0$	-1	$+2$
3.86×10^{-4}	$+1.411472 \times 10^{3}$	-2	$+5$	$+0$	-2
1.31×10^{-4}	$+2.270341 \times 10^{4}$	-1	$+3$	$+0$	-1
1.05×10^{-4}	-8.652321×10^{4}	-2	$+3$	$+0$	$+0$
9.92×10^{-5}	$+1.387493 \times 10^{3}$	-2	$+5$	$+1$	-3
8.06×10^{-5}	$+4.399535 \times 10^{4}$	$+0$	$+1$	$+0$	$+0$
6.45×10^{-5}	-4.255587×10^{4}	-2	$+4$	$+0$	-1
4.60×10^{-5}	-2.123995×10^{4}	-1	$+2$	-1	$+1$
4.28×10^{-5}	-2.128791×10^{4}	-1	$+2$	$+1$	-1
3.66×10^{-5}	-1.517825×10^{5}	-3	$+4$	$+0$	$+0$
3.49×10^{-5}	$+7.596451 \times 10^{1}$	$+0$	$+0$	-2	$+3$
3.45×10^{-5}	$+1.092546 \times 10^{5}$	$+1$	$+0$	$+0$	$+0$
2.54×10^{-5}	$+1.435452 \times 10^{3}$	-2	$+5$	-1	-1
2.01×10^{-5}	-1.078152×10^{5}	-3	$+5$	$+0$	-1
1.93×10^{-5}	-1.995139×10^{1}	$+0$	$+0$	$+2$	-1
1.85×10^{-5}	$+2.267943 \times 10^{4}$	-1	$+3$	$+1$	-2
1.82×10^{-5}	$+1.363514 \times 10^{3}$	-2	$+5$	$+2$	-4

$$z_5(t) \approx \sum_{j=1}^{N} \alpha_j \exp(i f_j t)$$

Quasi-periodic decomposition of $z_5 = e_5 \exp i\varpi_5$

$\lvert\alpha_j\rvert$	f_j ("/yr)	k_5	k_6	p_5	p_6
4.41×10^{-2}	$+4.027603 \times 10^{0}$	$+0$	$+0$	$+1$	$+0$
1.59×10^{-2}	$+2.800657 \times 10^{1}$	$+0$	$+0$	$+0$	$+1$
6.44×10^{-4}	-2.126393×10^{4}	-1	$+2$	$+0$	$+0$
6.28×10^{-4}	$+5.198554 \times 10^{1}$	$+0$	$+0$	-1	$+2$
3.86×10^{-4}	$+1.411472 \times 10^{3}$	-2	$+5$	$+0$	-2
1.31×10^{-4}	$+2.270341 \times 10^{4}$	-1	$+3$	$+0$	-1
1.05×10^{-4}	-8.652321×10^{4}	-2	$+3$	$+0$	$+0$
9.92×10^{-5}	$+1.387493 \times 10^{3}$	-2	$+5$	$+1$	-3
8.06×10^{-5}	$+4.399535 \times 10^{4}$	$+0$	$+1$	$+0$	$+0$
6.45×10^{-5}	-4.255587×10^{4}	-2	$+4$	$+0$	-1
4.60×10^{-5}	-2.123995×10^{4}	-1	$+2$	-1	$+1$
4.28×10^{-5}	-2.128791×10^{4}	-1	$+2$	$+1$	-1
3.66×10^{-5}	-1.517825×10^{5}	-3	$+4$	$+0$	$+0$
3.49×10^{-5}	$+7.596451 \times 10^{1}$	$+0$	$+0$	-2	$+3$
3.45×10^{-5}	$+1.092546 \times 10^{5}$	$+1$	$+0$	$+0$	$+0$
2.54×10^{-5}	$+1.435452 \times 10^{3}$	-2	$+5$	-1	-1
2.01×10^{-5}	-1.078152×10^{5}	-3	$+5$	$+0$	-1
1.93×10^{-5}	-1.995139×10^{1}	$+0$	$+0$	$+2$	-1
1.85×10^{-5}	$+2.267943 \times 10^{4}$	-1	$+3$	$+1$	-2
1.82×10^{-5}	$+1.363514 \times 10^{3}$	-2	$+5$	$+2$	-4

$$z_5(t) \approx \sum_{j=1}^{N} \alpha_j \exp(i f_j t)$$

$$n_5 - 2n_6$$

$$\frac{2}{5}(n_5 - 2n_6) \approx 8500\text{"}/yr$$

$$\nu \in [7500, 9200]\text{"}/yr$$

Family II

Quasi-periodic decomposition of $z_5 = e_5 \exp i\varpi_5$

$\lvert\alpha_j\rvert$	f_j ("/yr)	k_5	k_6	p_5	p_6
4.41×10^{-2}	$+4.027603 \times 10^{0}$	+0	+0	+1	+0
1.59×10^{-2}	$+2.800657 \times 10^{1}$	+0	+0	+0	+1
6.44×10^{-4}	-2.126393×10^{4}	−1	+2	+0	+0
6.28×10^{-4}	$+5.198554 \times 10^{1}$	+0	+0	−1	+2
3.86×10^{-4}	$+1.411472 \times 10^{3}$	−2	+5	+0	−2
1.31×10^{-4}	$+2.270341 \times 10^{4}$	−1	+3	+0	−1
1.05×10^{-4}	-8.652321×10^{4}	−2	+3	+0	+0
9.92×10^{-5}	$+1.387493 \times 10^{3}$	−2	+5	+1	−3
8.06×10^{-5}	$+4.399535 \times 10^{4}$	+0	+1	+0	+0
6.45×10^{-5}	-4.255587×10^{4}	−2	+4	+0	−1
4.60×10^{-5}	-2.123995×10^{4}	−1	+2	−1	+1
4.28×10^{-5}	-2.128791×10^{4}	−1	+2	+1	−1
3.66×10^{-5}	-1.517825×10^{5}	−3	+4	+0	+0
3.49×10^{-5}	$+7.596451 \times 10^{1}$	+0	+0	−2	+3
3.45×10^{-5}	$+1.092546 \times 10^{5}$	+1	+0	+0	+0
2.54×10^{-5}	$+1.435452 \times 10^{3}$	−2	+5	−1	−1
2.01×10^{-5}	-1.078152×10^{5}	−3	+5	+0	−1
1.93×10^{-5}	-1.995139×10^{1}	+0	+0	+2	−1
1.85×10^{-5}	$+2.267943 \times 10^{4}$	−1	+3	+1	−2
1.82×10^{-5}	$+1.363514 \times 10^{3}$	−2	+5	+2	−4

$$z_5(t) \approx \sum_{j=1}^{N} \alpha_j \exp(i f_j t)$$

$n_5 - 2n_6$

$2n_5 - 5n_6\ (+2g_6)$

$$\frac{2}{5}(n_5 - 2n_6) \approx 8500\text{"}/yr$$

$$\nu \in [7500, 9200]\text{"}/yr$$

Family II

$$\frac{2n_5 - 5n_6}{4} \approx 350\text{"}/yr$$

$$g \in [230, 450]\text{"}/yr$$

Family IV

4 different families of resonances

4 different families of resonances

Secondary resonances

Famille I: $p\nu - n_5 + qg + q_5 g_5 + q_6 g_6 = 0$

Famille II: $5\nu - 2(n_5 - 2n_6) + pg + p_5 g_5 + p_6 g_6 = 0$

4 different families of resonances

Secondary resonances

Famille I: $p\nu - n_5 + qg + q_5 g_5 + q_6 g_6 = 0$

Famille II: $5\nu - 2(n_5 - 2n_6) + pg + p_5 g_5 + p_6 g_6 = 0$

Secular resonances

Famille III: $q\,s + q_6 s_6 + p_5 g_5 + p_6 g_6 = 0$

4 different families of resonances

Secondary resonances

Famille I: $p\nu - n_5 + qg + q_5 g_5 + q_6 g_6 = 0$

Famille II: $5\nu - 2(n_5 - 2n_6) + pg + p_5 g_5 + p_6 g_6 = 0$

Secular resonances

Famille III: $q\,s + q_6 s_6 + p_5 g_5 + p_6 g_6 = 0$

G. I. + secular frequencies

Famille IV: $pg - (2n_5 - 5n_6) + p_5 g_5 + p_6 g_6 = 0$

4 different families of resonances

Secondary resonances

Famille I: $p\nu - n_5 + qg + q_5 g_5 + q_6 g_6 = 0$

Famille II: $5\nu - 2(n_5 - 2n_6) + pg + p_5 g_5 + p_6 g_6 = 0$

Secular resonances

Famille III: $q\,s + q_6 s_6 + p_5 g_5 + p_6 g_6 = 0$

G. I. + secular frequencies

Famille IV: $pg - (2n_5 - 5n_6) + p_5 g_5 + p_6 g_6 = 0$

4 different families of resonances

Secondary resonances

Famille I: $p\nu - n_5 + qg + q_5 g_5 + q_6 g_6 = 0$

Famille II: $5\nu - 2(n_5 - 2n_6) + pg + p_5 g_5 + p_6 g_6 = 0$

Secular resonances

Famille III: $q\,s + q_6 s_6 + p_5 g_5 + p_6 g_6 = 0$

G. I. + secular frequencies

Famille IV: $pg - (2n_5 - 5n_6) + p_5 g_5 + p_6 g_6 = 0$

4 different families of resonances

Secondary resonances

Famille I: $p\nu - n_5 + qg + q_5 g_5 + q_6 g_6 = 0$

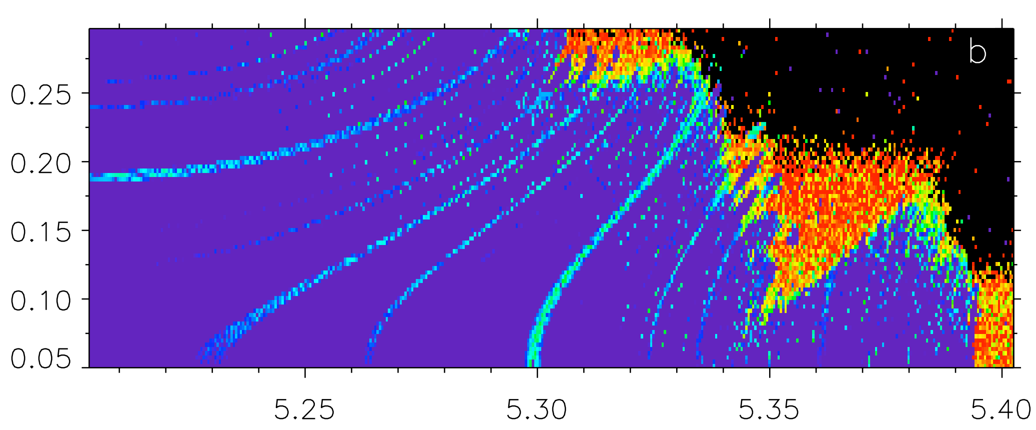
Famille II: $5\nu - 2(n_5 - 2n_6) + pg + p_5 g_5 + p_6 g_6 = 0$

Secular resonances

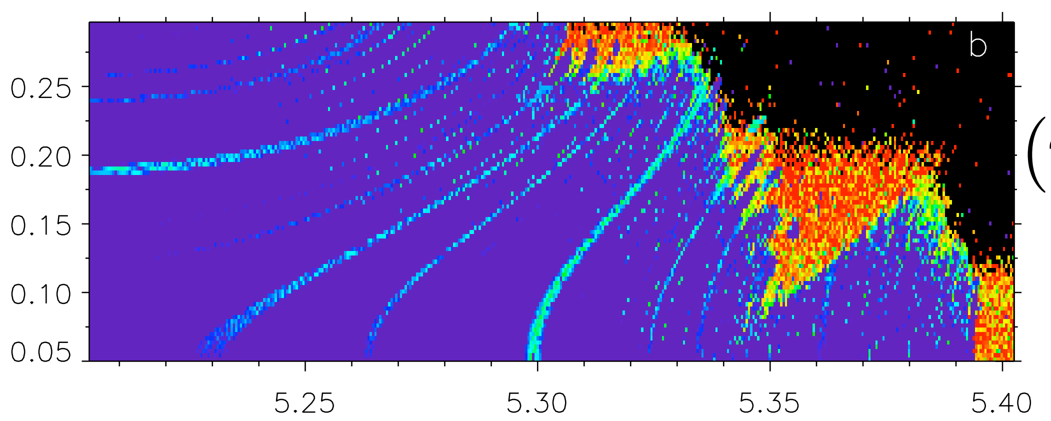
Famille III: $q\,s + q_6 s_6 + p_5 g_5 + p_6 g_6 = 0$

G. I. + secular frequencies

Famille IV: $pg - (2n_5 - 5n_6) + p_5 g_5 + p_6 g_6 = 0$

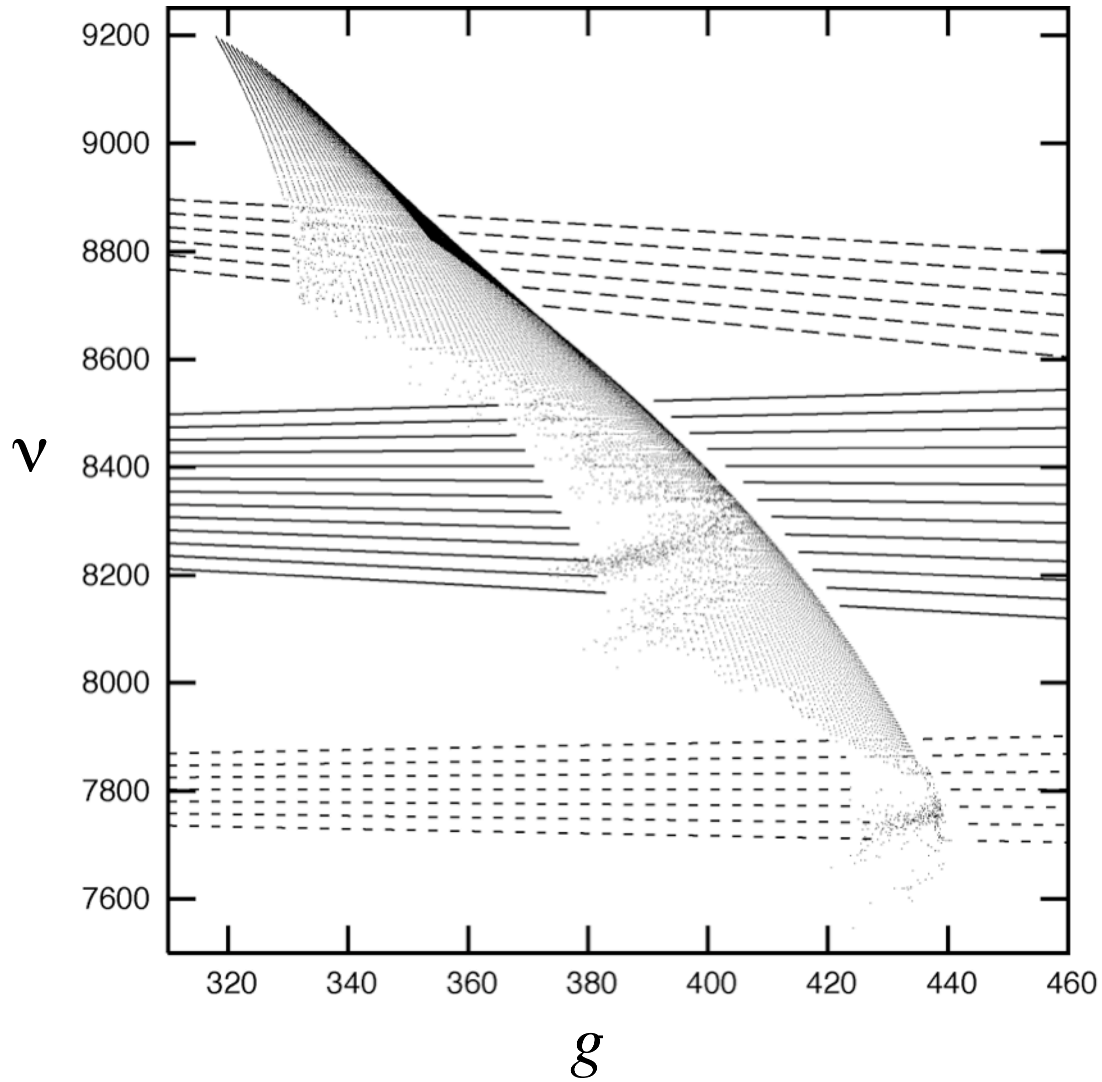


E.R.T.B.P (S+J+T)



E.R.T.B.P (S+J+T)

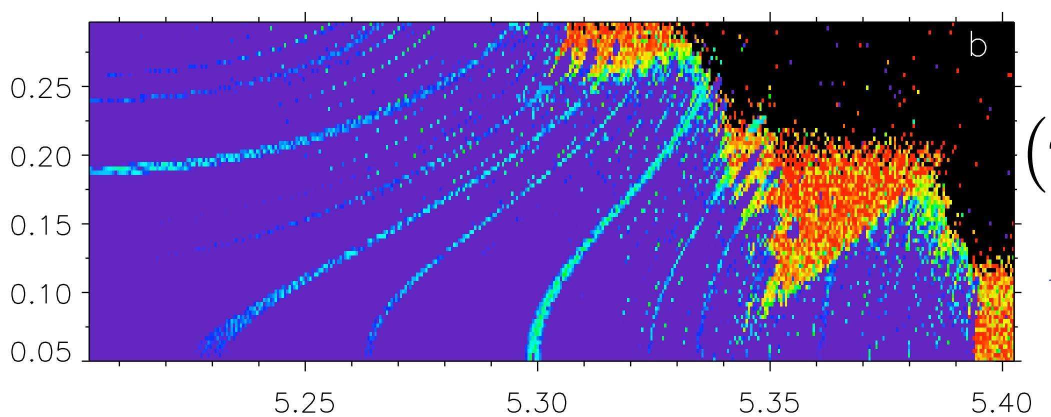
$$(\nu, g, s) + n_5$$



$12\nu - n_5 + qg = 0$ avec $q \in \{8, \cdots, 13\}$

$13\nu - n_5 + qg = 0$ avec $q \in \{-4, \cdots, 8\}$

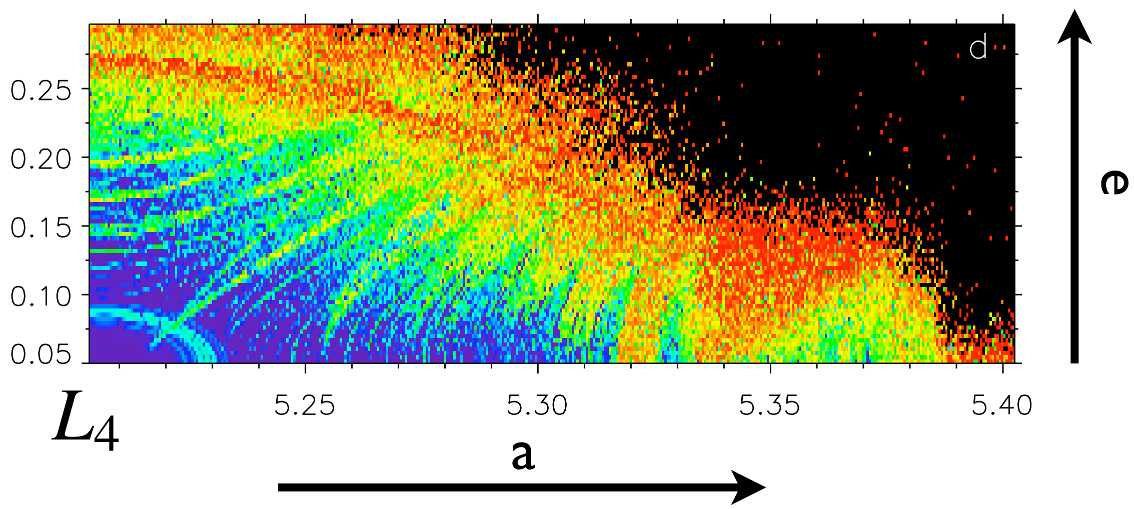
$14\nu - n_5 + qg = 0$ avec $q \in \{-3, \cdots, 3\}$

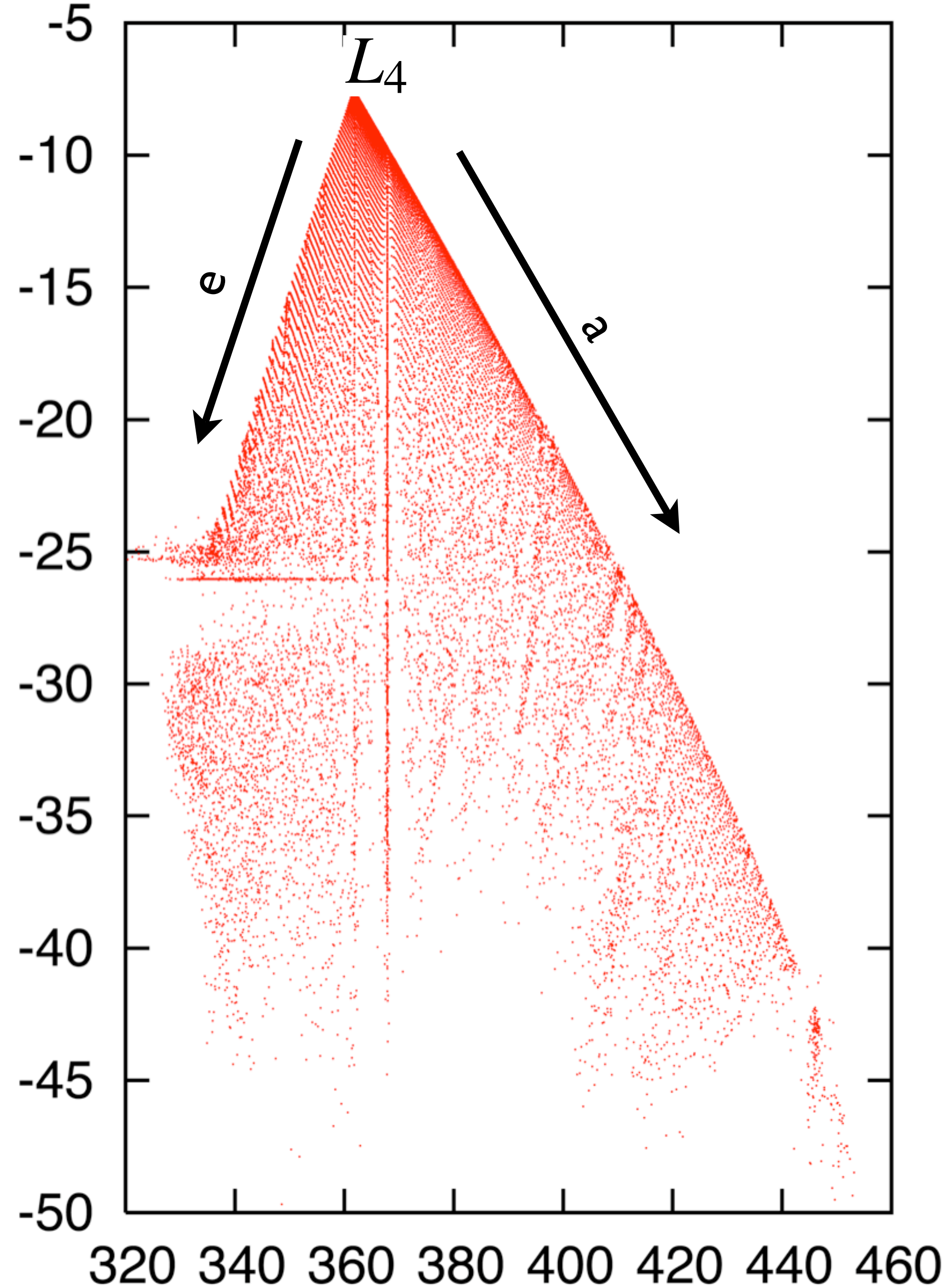


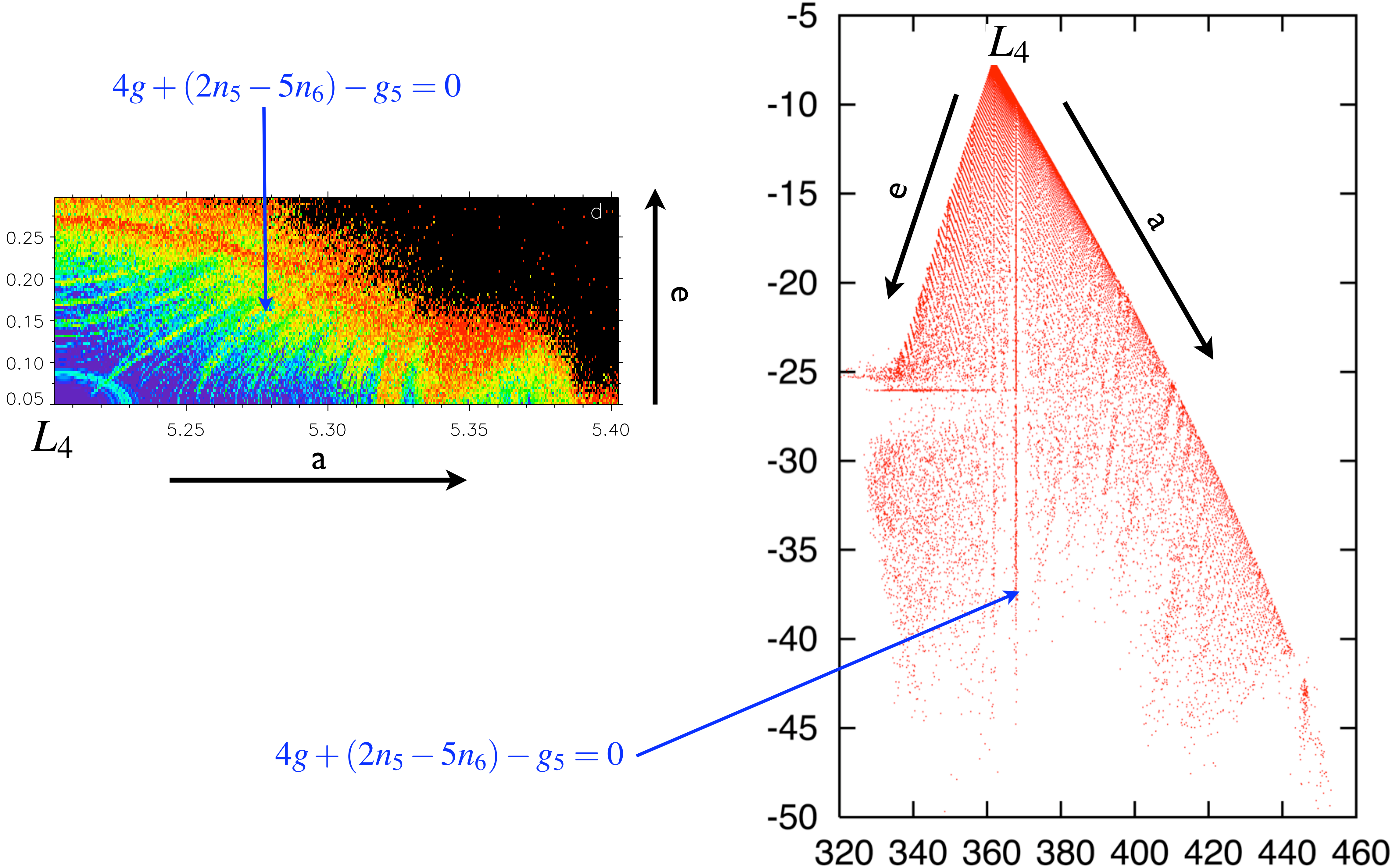
E.R.T.B.P (S+J+T)

$(\nu, g, s) + n_5$

$p\nu - n_5 + qg = 0$


d
0.25
0.20
0.15
0.10
0.05
L_4
5.25
5.30
5.35
5.40
a
e

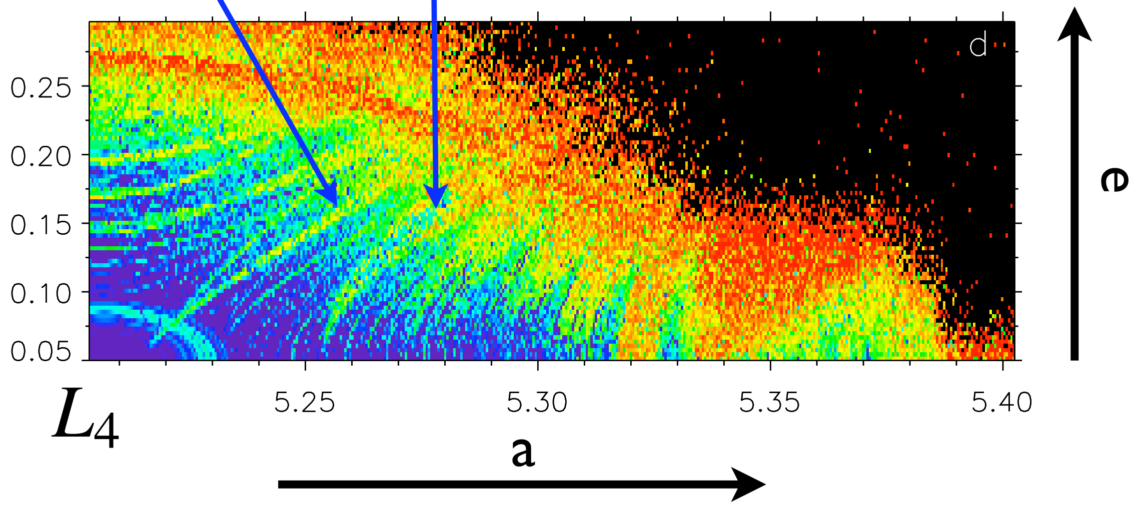

-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
320 340 360 380 400 420 440 460
L_4
e
a

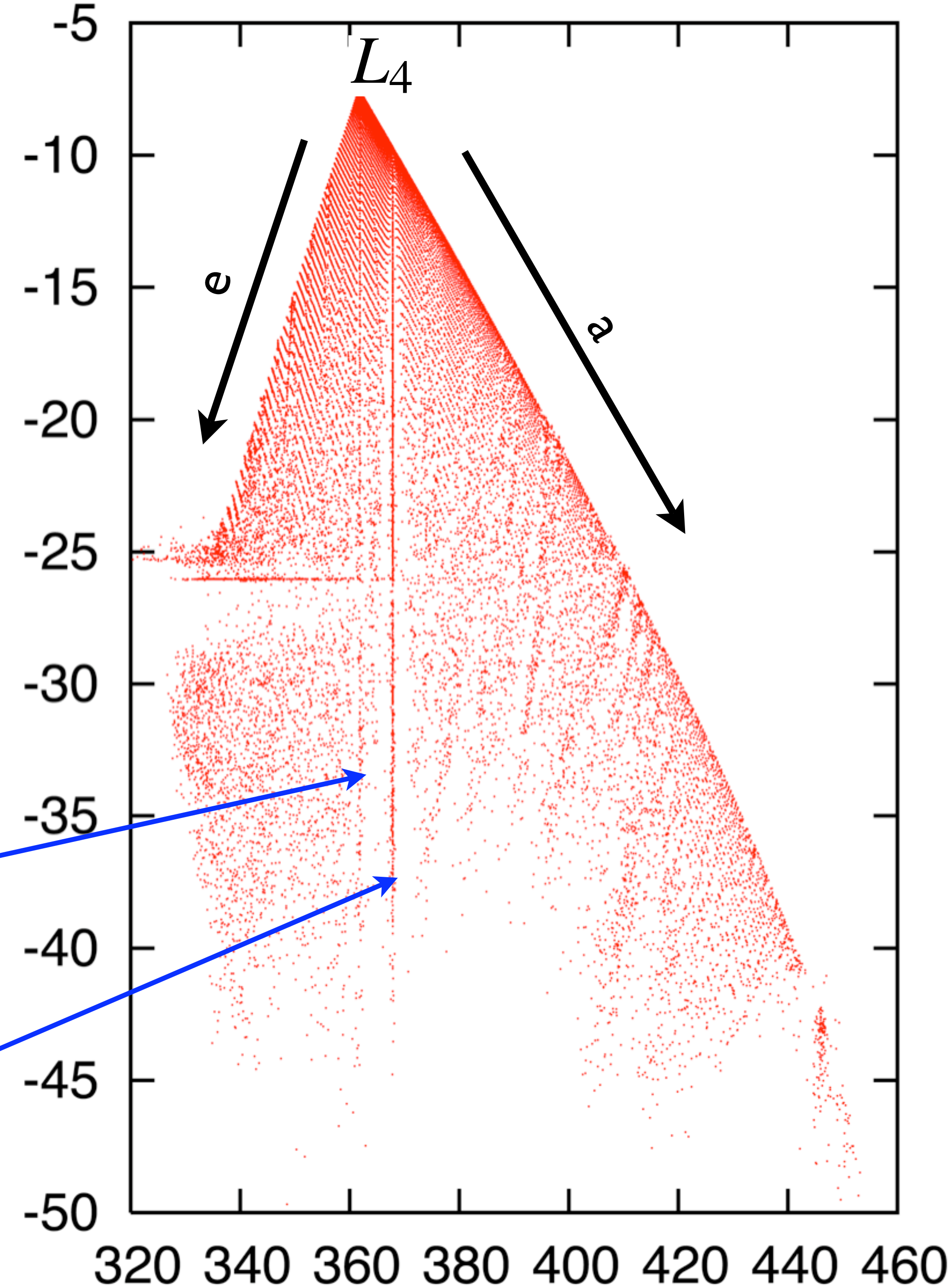


$4g + (2n_5 - 5n_6) - g_5 = 0$
d
0.25
0.20
0.15
0.10
0.05
L_4
5.25
5.30
5.35
5.40
a
e
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
320 340 360 380 400 420 440 460
L_4
e
a
$4g + (2n_5 - 5n_6) - g_5 = 0$

$$4g + (2n_5 - 5n_6) - 2g_5 + g_6 = 0$$

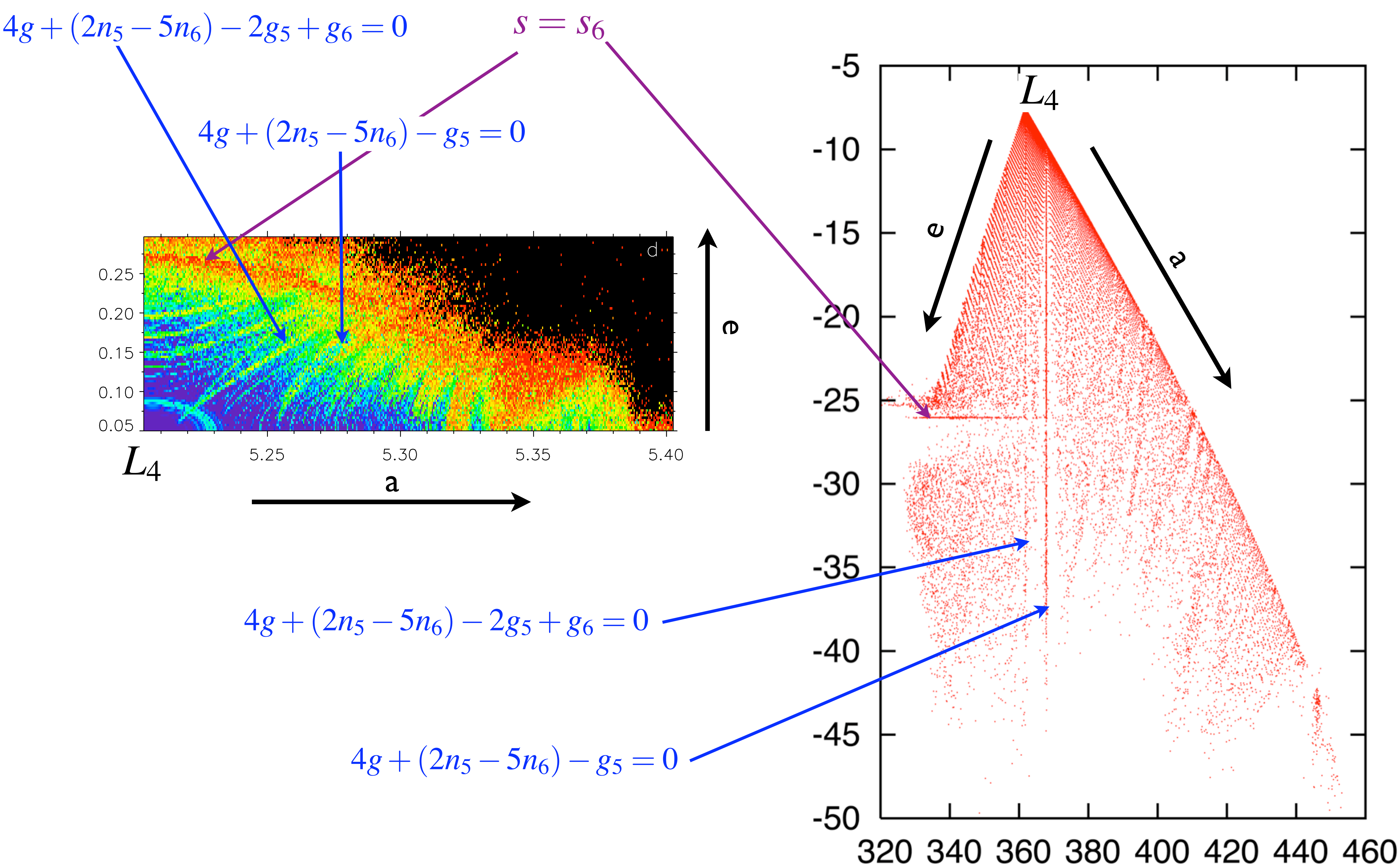
$$4g + (2n_5 - 5n_6) - g_5 = 0$$

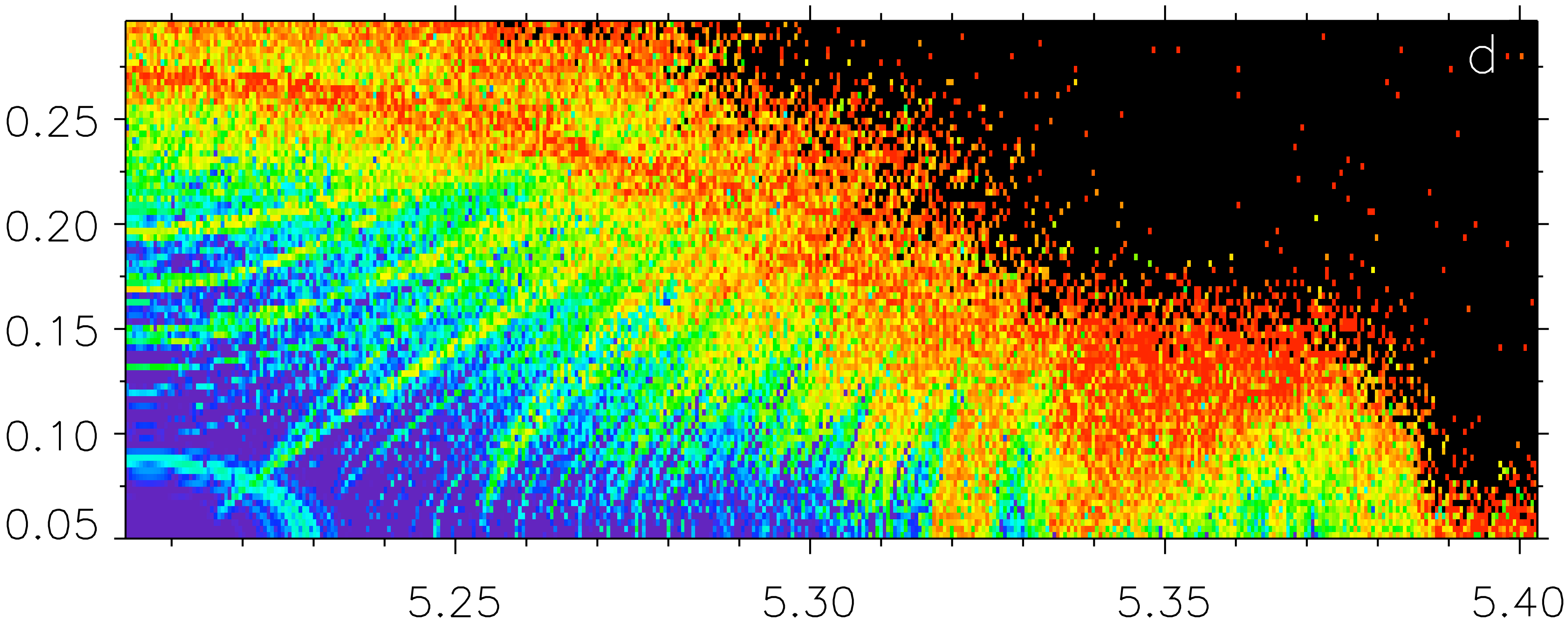




$$4g + (2n_5 - 5n_6) - 2g_5 + g_6 = 0$$

$$4g + (2n_5 - 5n_6) - g_5 = 0$$

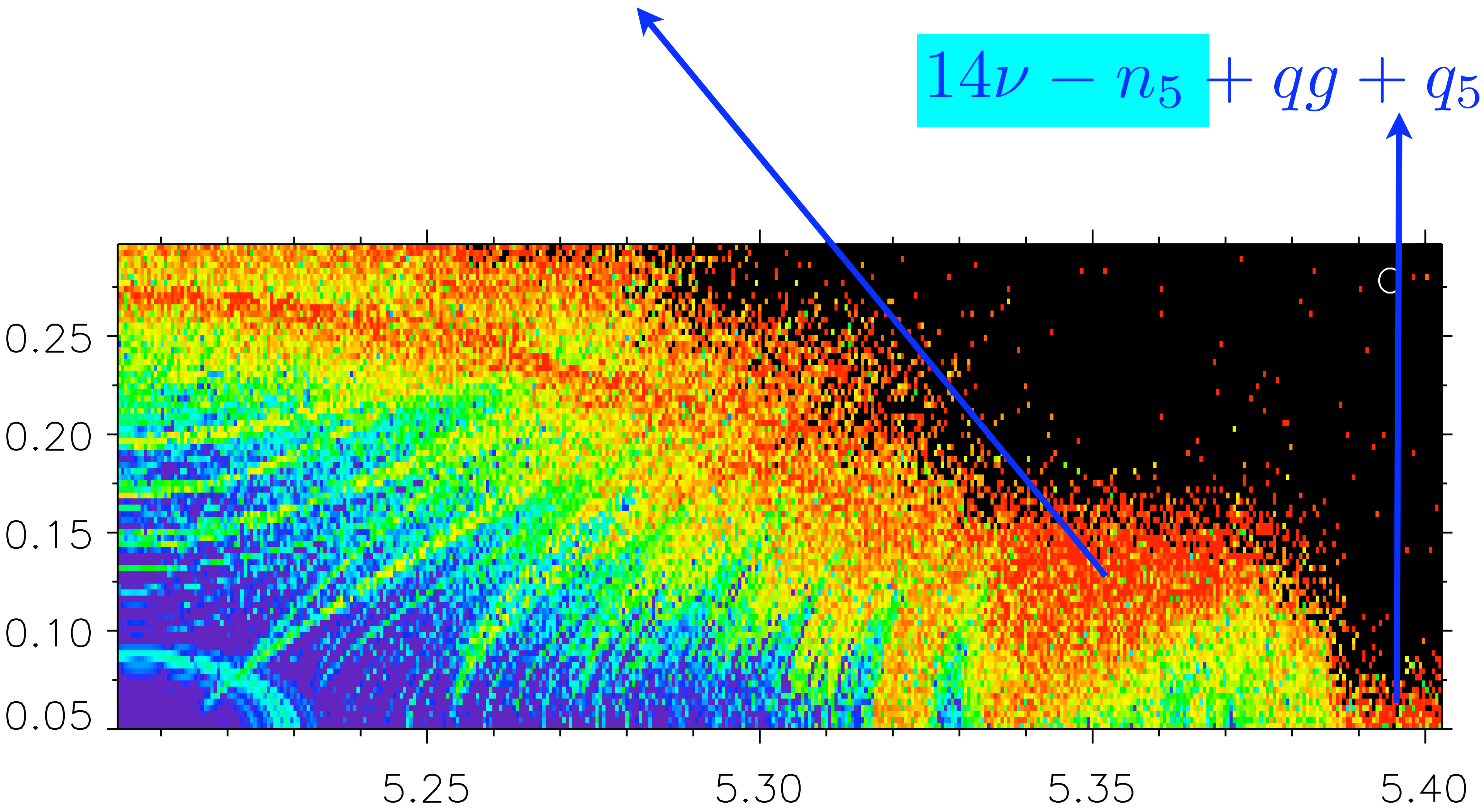

$4g+(2n_5-5n_6)-2g_5+g_6=0$
$s=s_6$
$4g+(2n_5-5n_6)-g_5=0$
d
0.25
0.20
0.15
0.10
0.05
L_4
5.25
5.30
5.35
5.40
a
e
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
320
340
360
380
400
420
440
460
L_4
e
a
$4g+(2n_5-5n_6)-2g_5+g_6=0$
$4g+(2n_5-5n_6)-g_5=0$



d
0.25
0.20
0.15
0.10
0.05
5.25
5.30
5.35
5.40

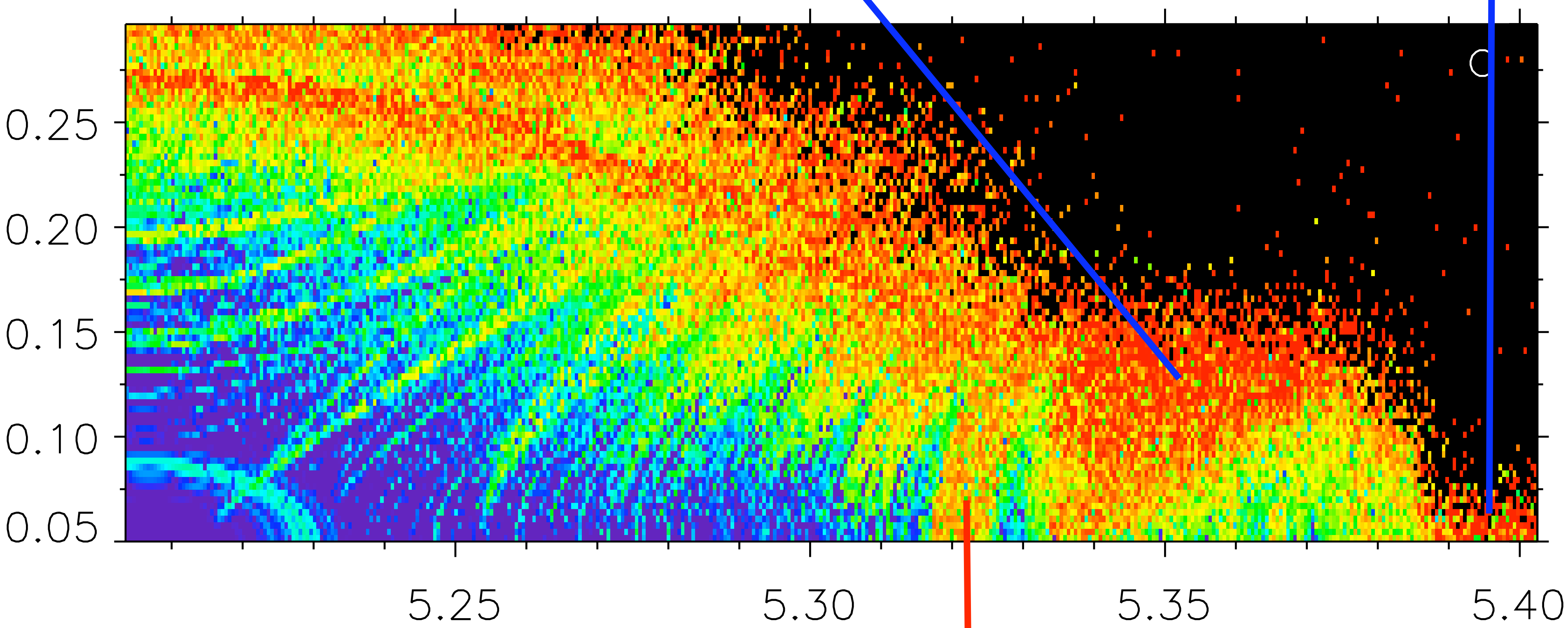
$$13\nu - n_5 + qg + q_5 g_5 + q_6 g_6 = 0$$

$$14\nu - n_5 + qg + q_5 g_5 + q_6 g_6 = 0$$



$$13\nu - n_5 + qg + q_5 g_5 + q_6 g_6 = 0$$

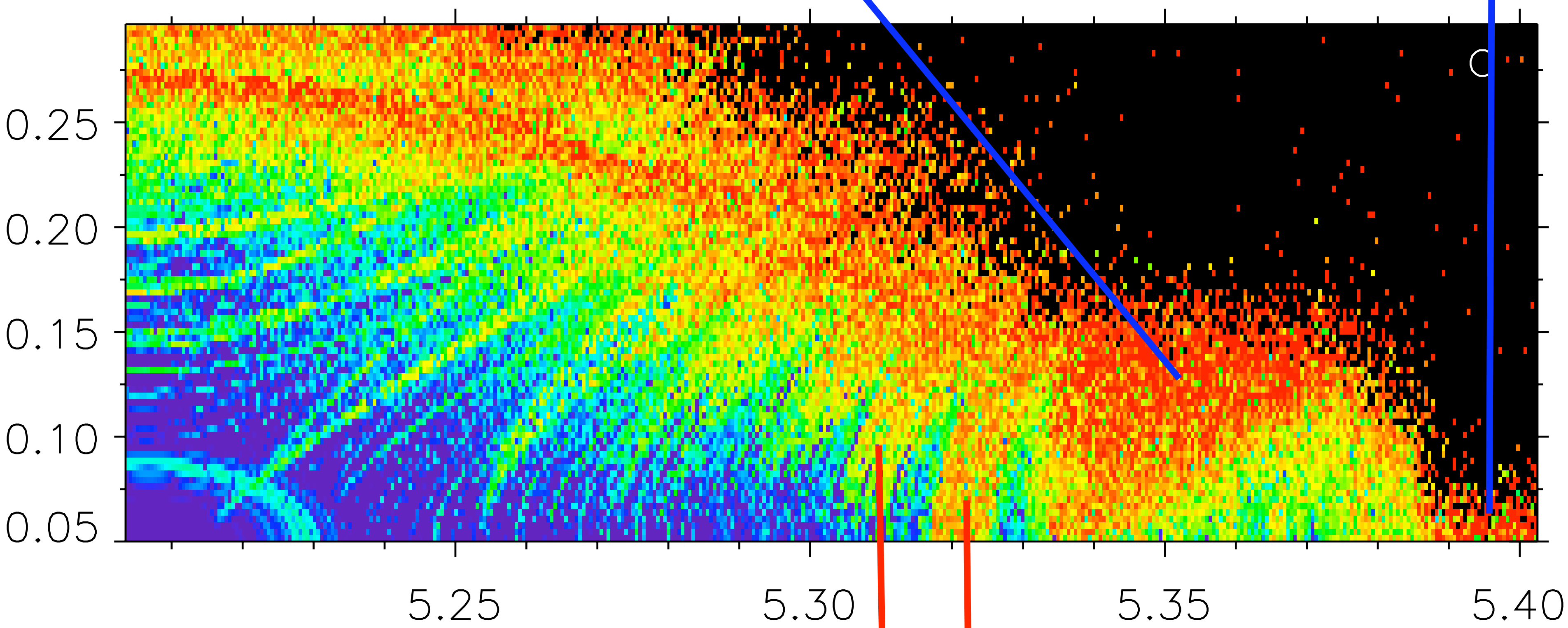
$$14\nu - n_5 + qg + q_5 g_5 + q_6 g_6 = 0$$



$$5\nu - 2(n_5 - 2n_6) - 0g + p_5 g_5 + p_6 g_6 = 0$$

$$13\nu - n_5 + qg + q_5 g_5 + q_6 g_6 = 0$$

$$14\nu - n_5 + qg + q_5 g_5 + q_6 g_6 = 0$$

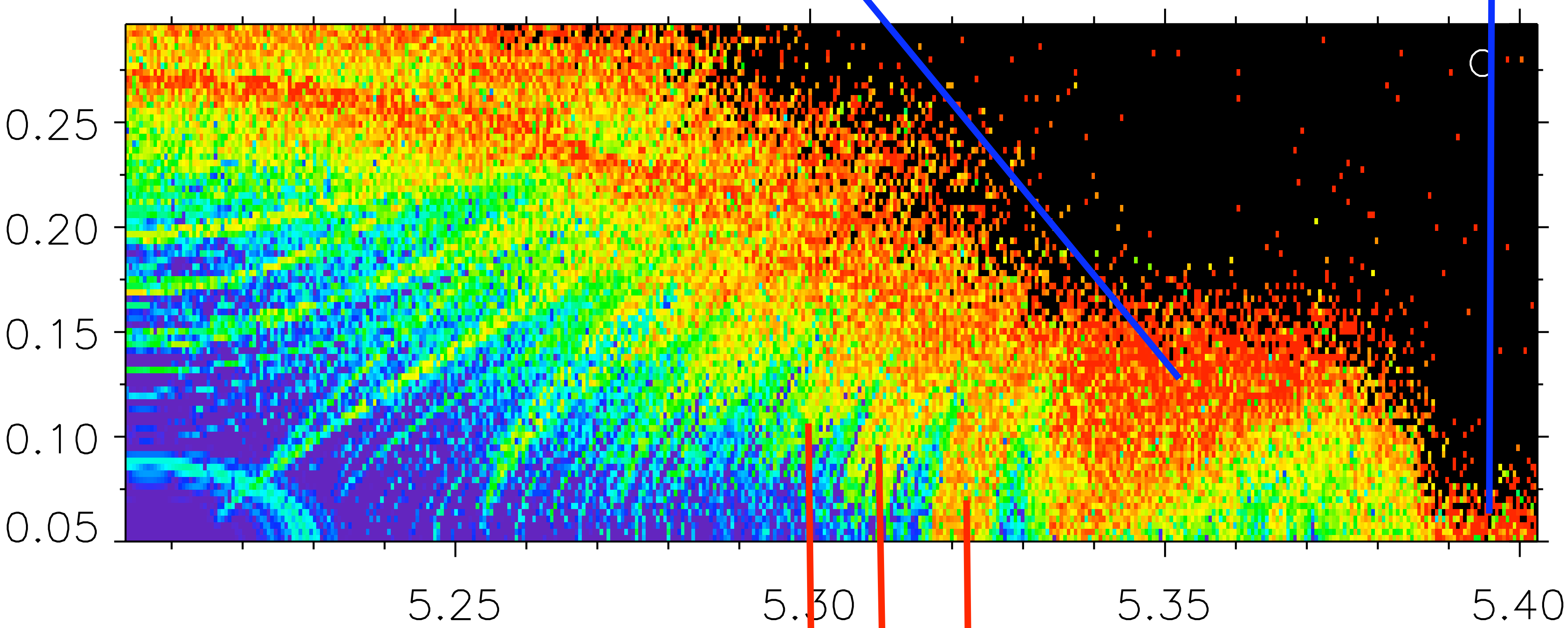


$$5\nu - 2(n_5 - 2n_6) - 0g + p_5 g_5 + p_6 g_6 = 0$$

$$5\nu - 2(n_5 - 2n_6) - 1g + p_5 g_5 + p_6 g_6 = 0$$

$$13\nu - n_5 + qg + q_5g_5 + q_6g_6 = 0$$

$$14\nu - n_5 + qg + q_5g_5 + q_6g_6 = 0$$

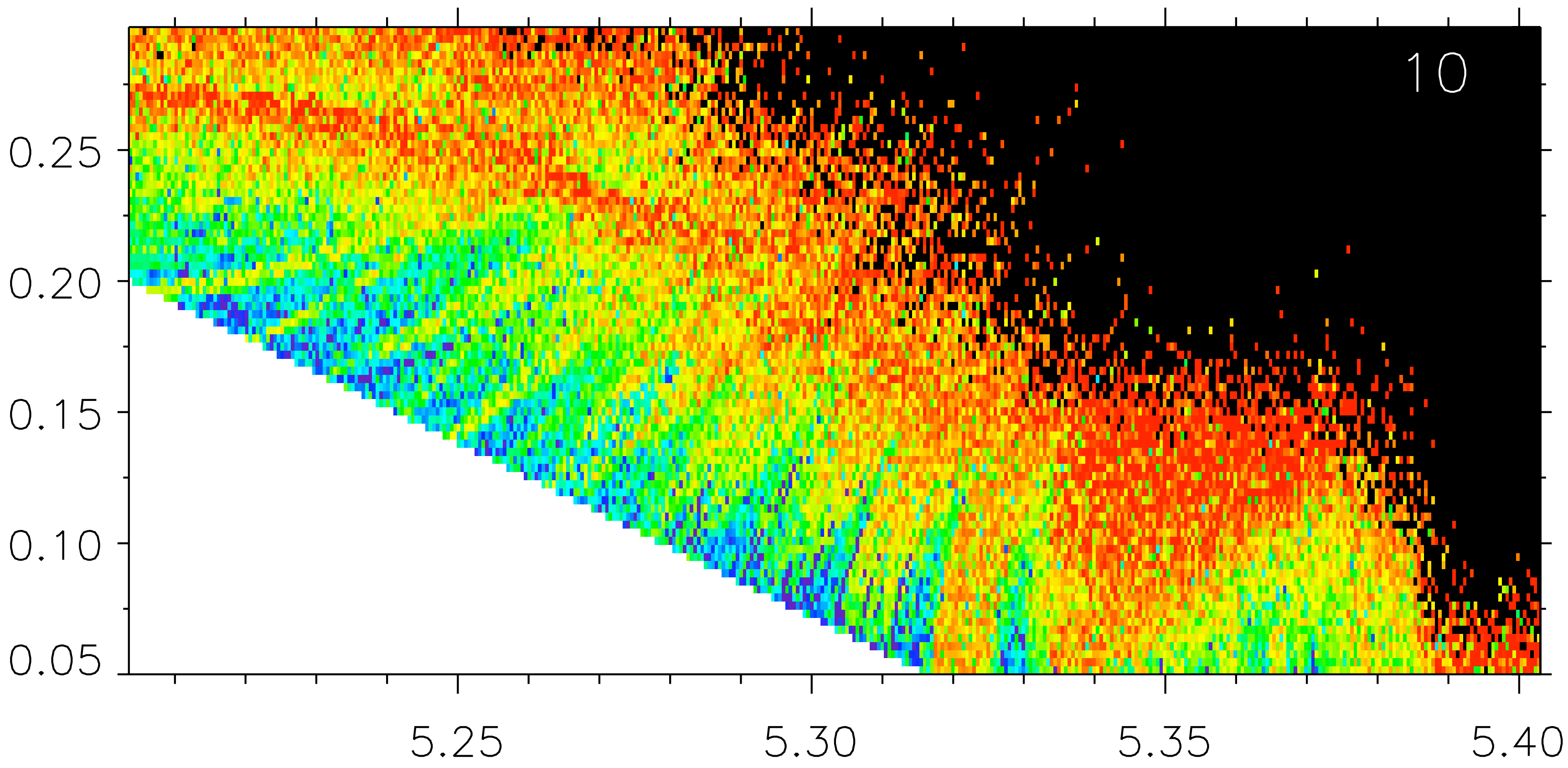


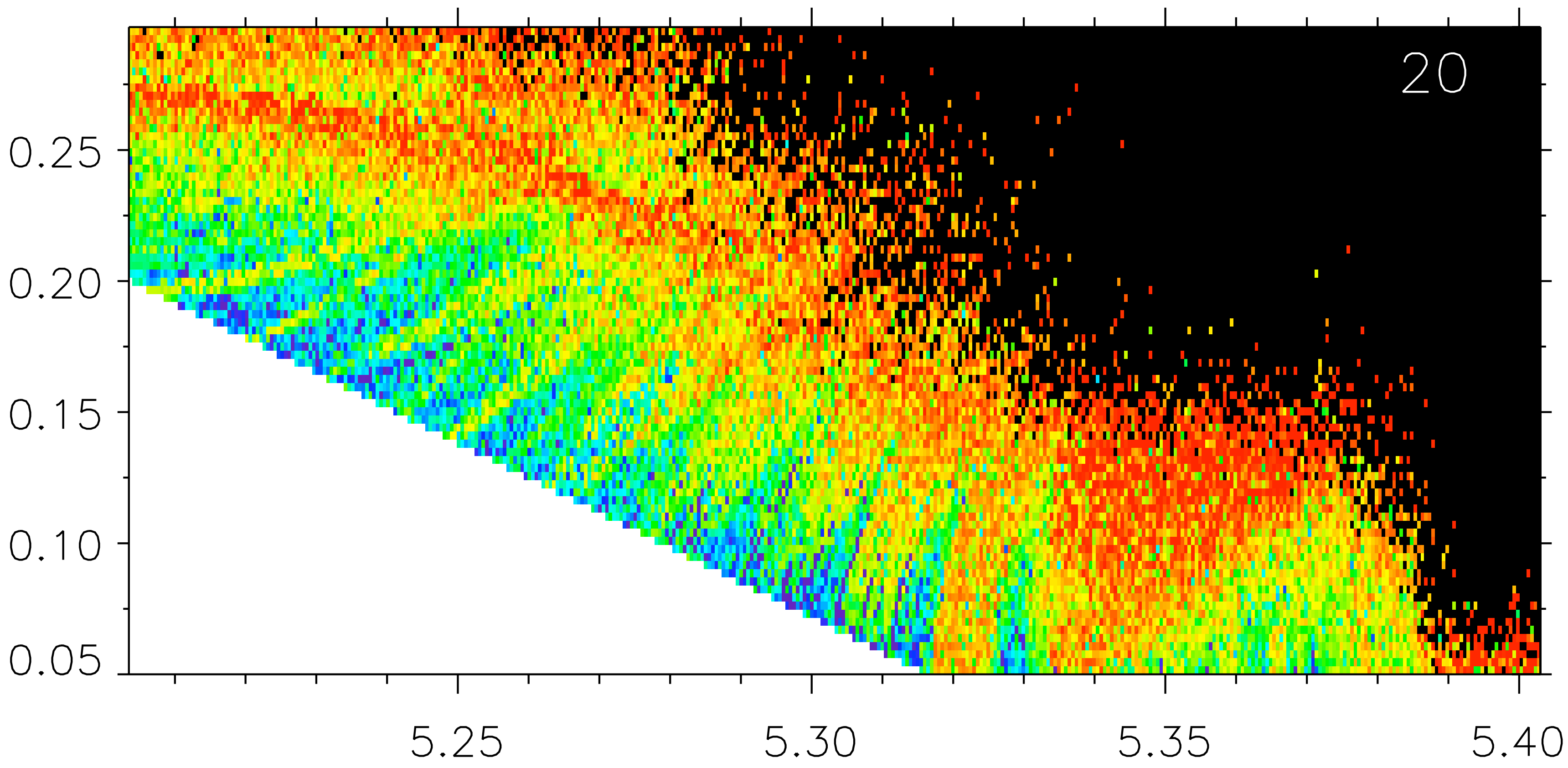
$$5\nu - 2(n_5 - 2n_6) - 0g + p_5g_5 + p_6g_6 = 0$$

$$5\nu - 2(n_5 - 2n_6) - 1g + p_5g_5 + p_6g_6 = 0$$

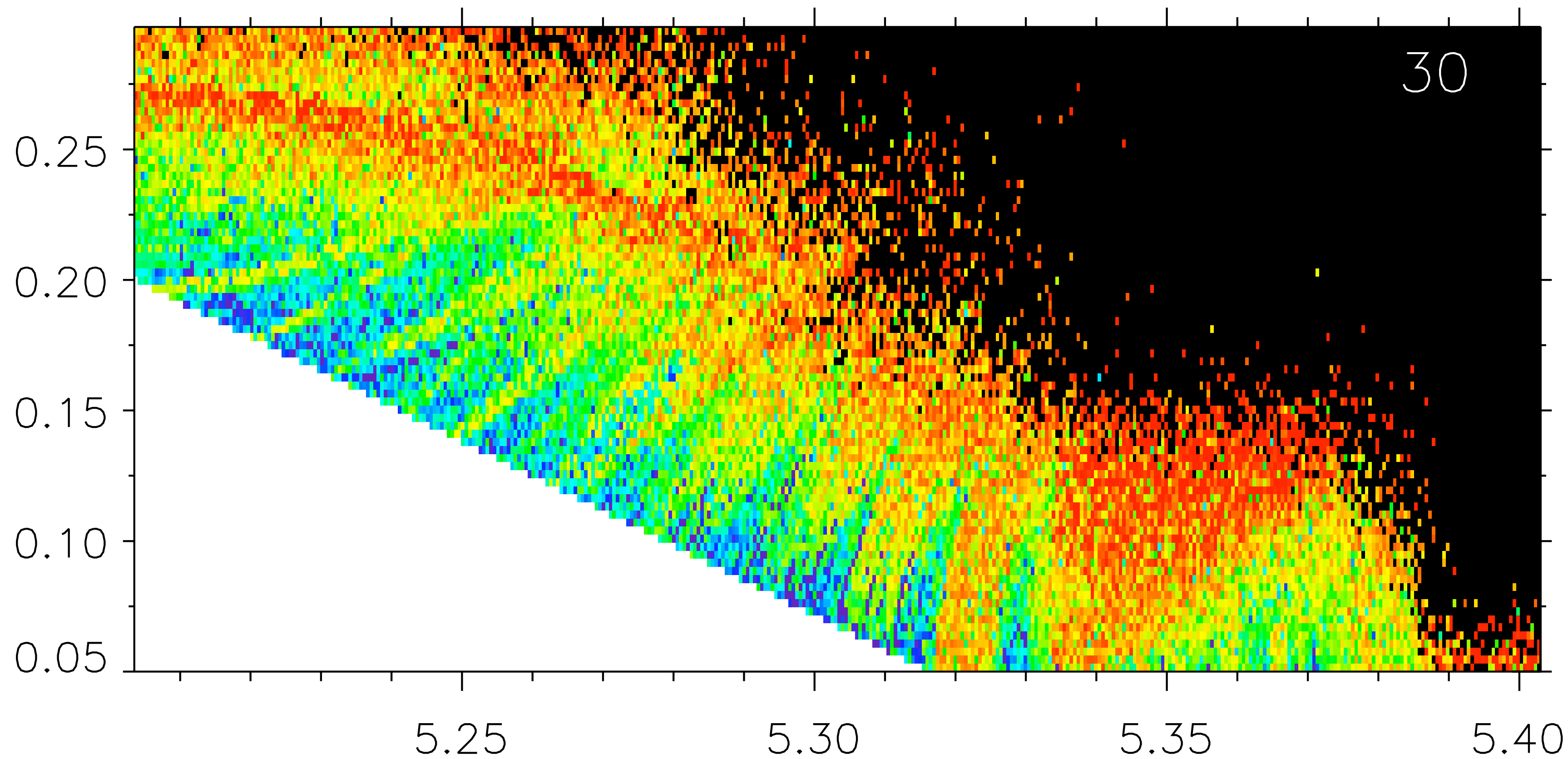
$$5\nu - 2(n_5 - 2n_6) - 2g + p_5g_5 + p_6g_6 = 0$$

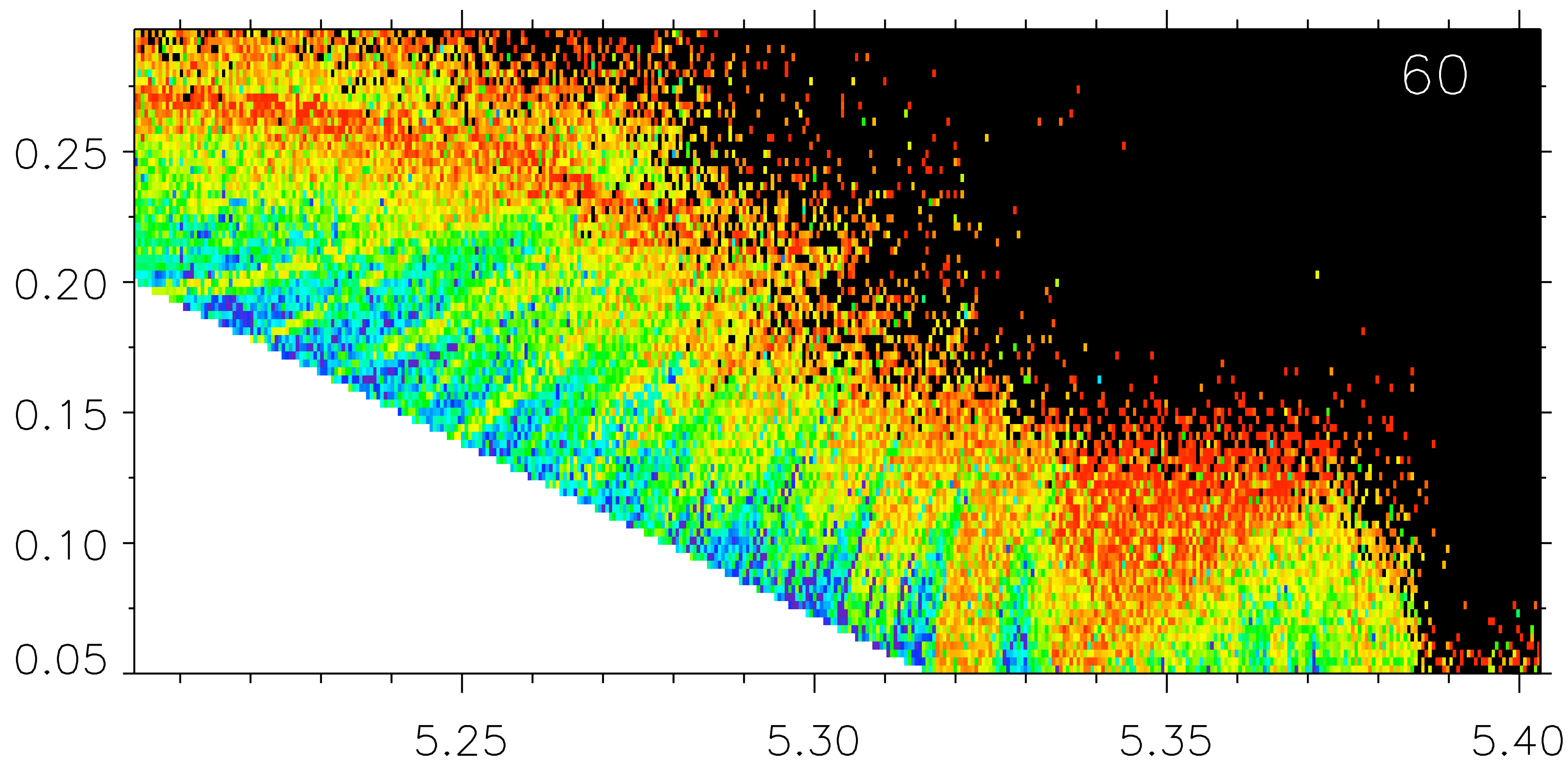
Long-term stability

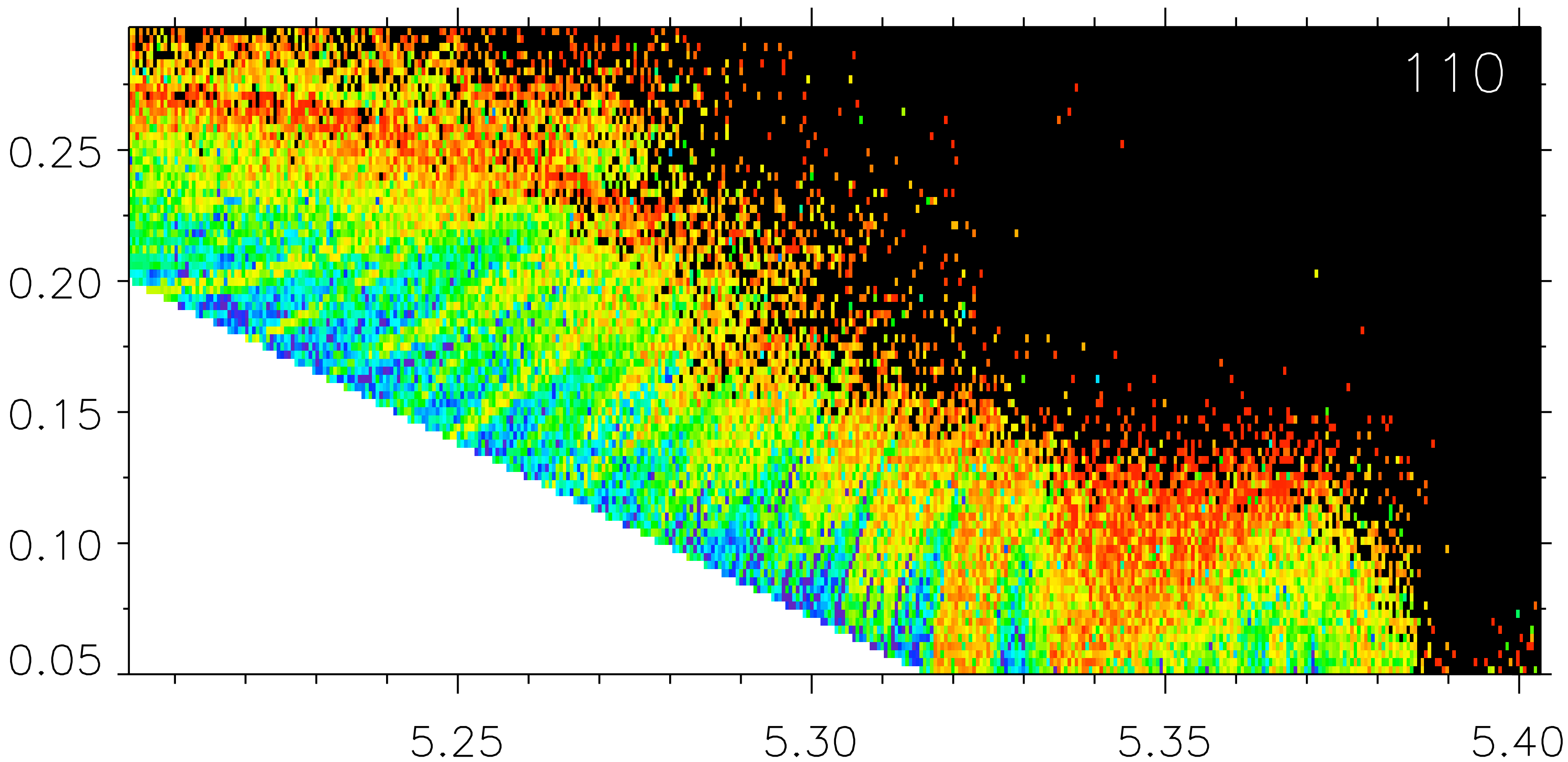




20
0.25
0.20
0.15
0.10
0.05
5.25
5.30
5.35
5.40


30
0.25
0.20
0.15
0.10
0.05
5.25
5.30
5.35
5.40


60
0.25
0.20
0.15
0.10
0.05
5.25
5.30
5.35
5.40


110
0.25
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0.15
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0.05
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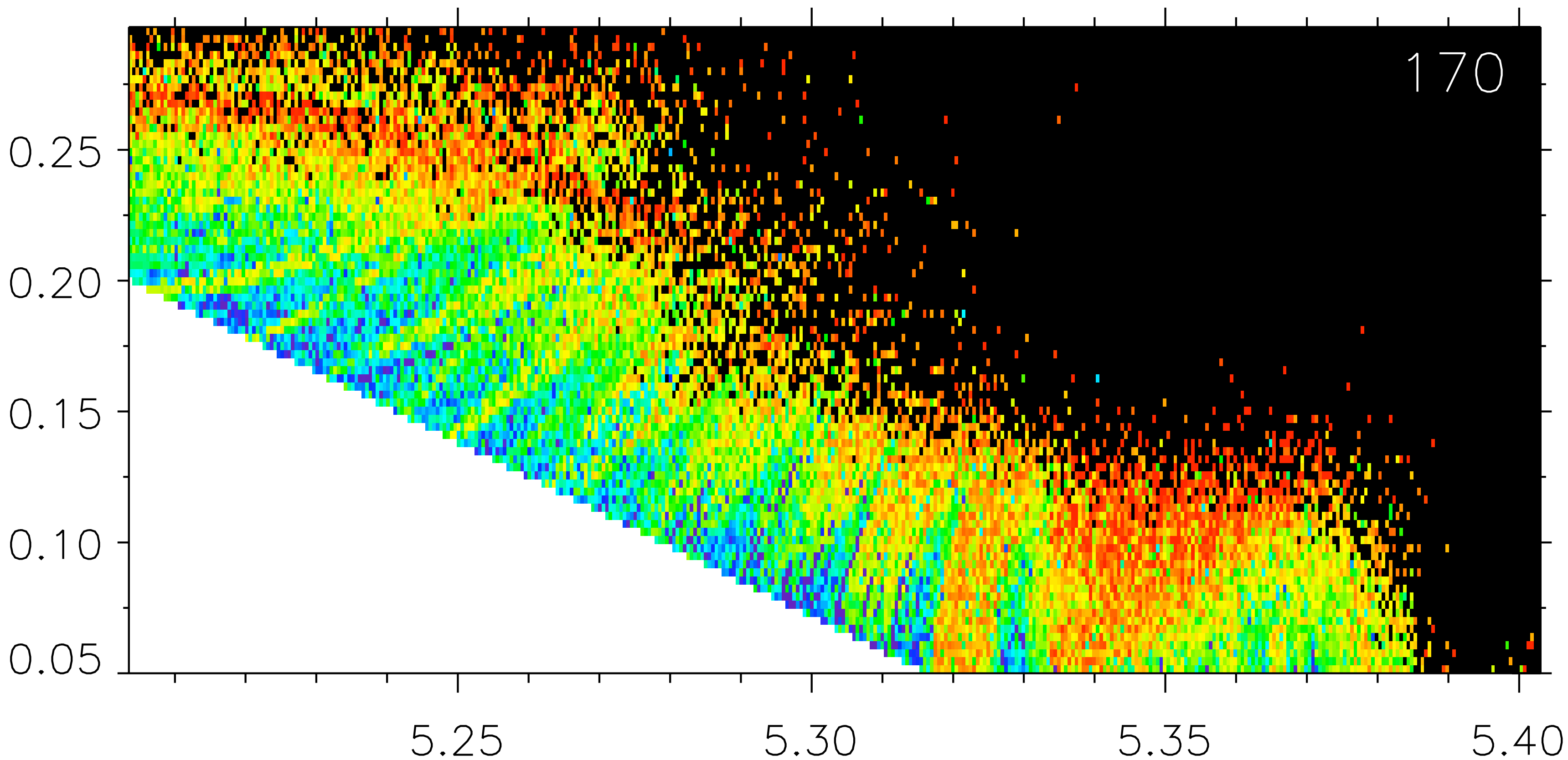
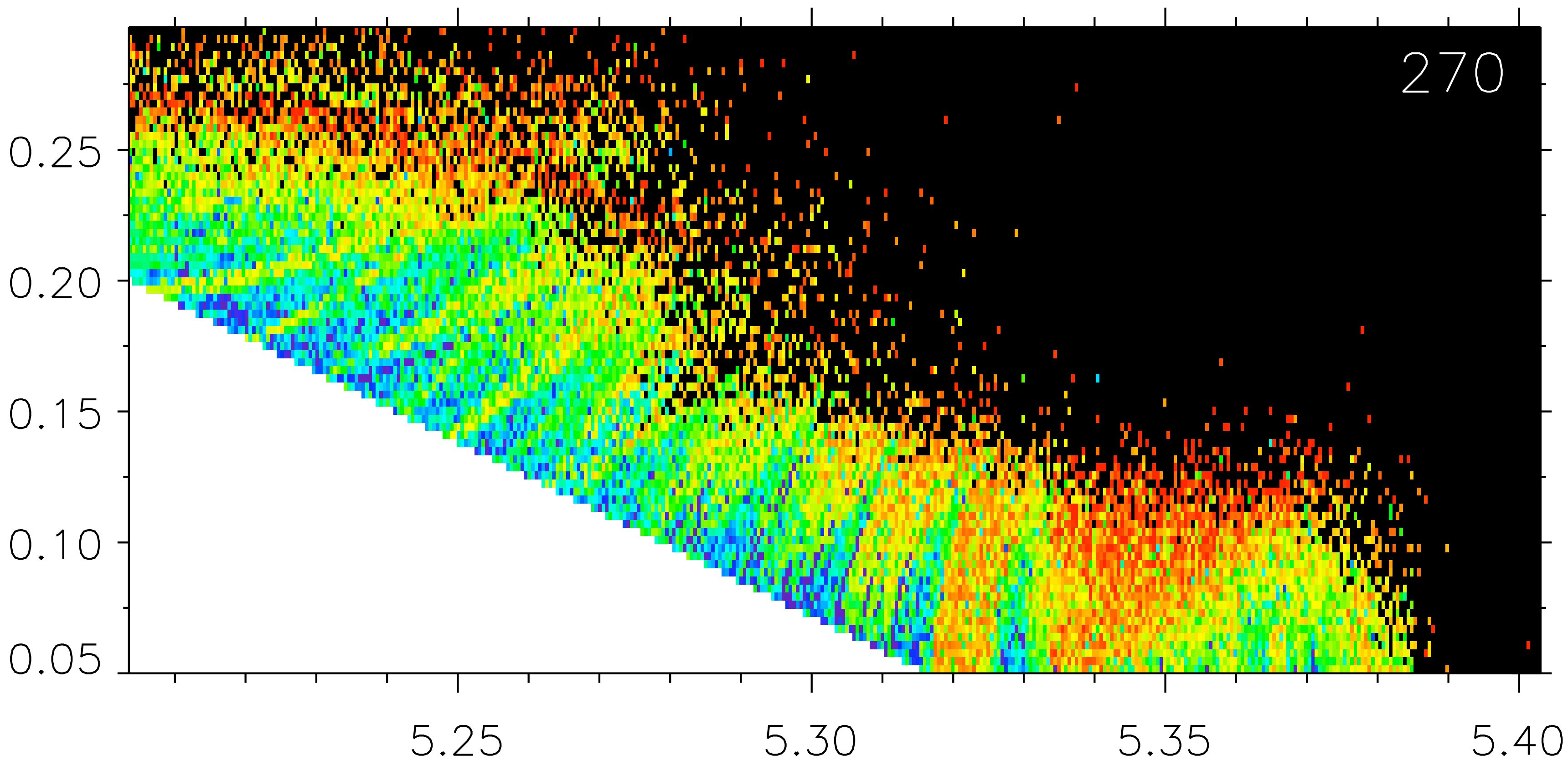
5.30
5.35
5.40



170
0.25
0.20
0.15
0.10
0.05
5.25
5.30
5.35
5.40



270
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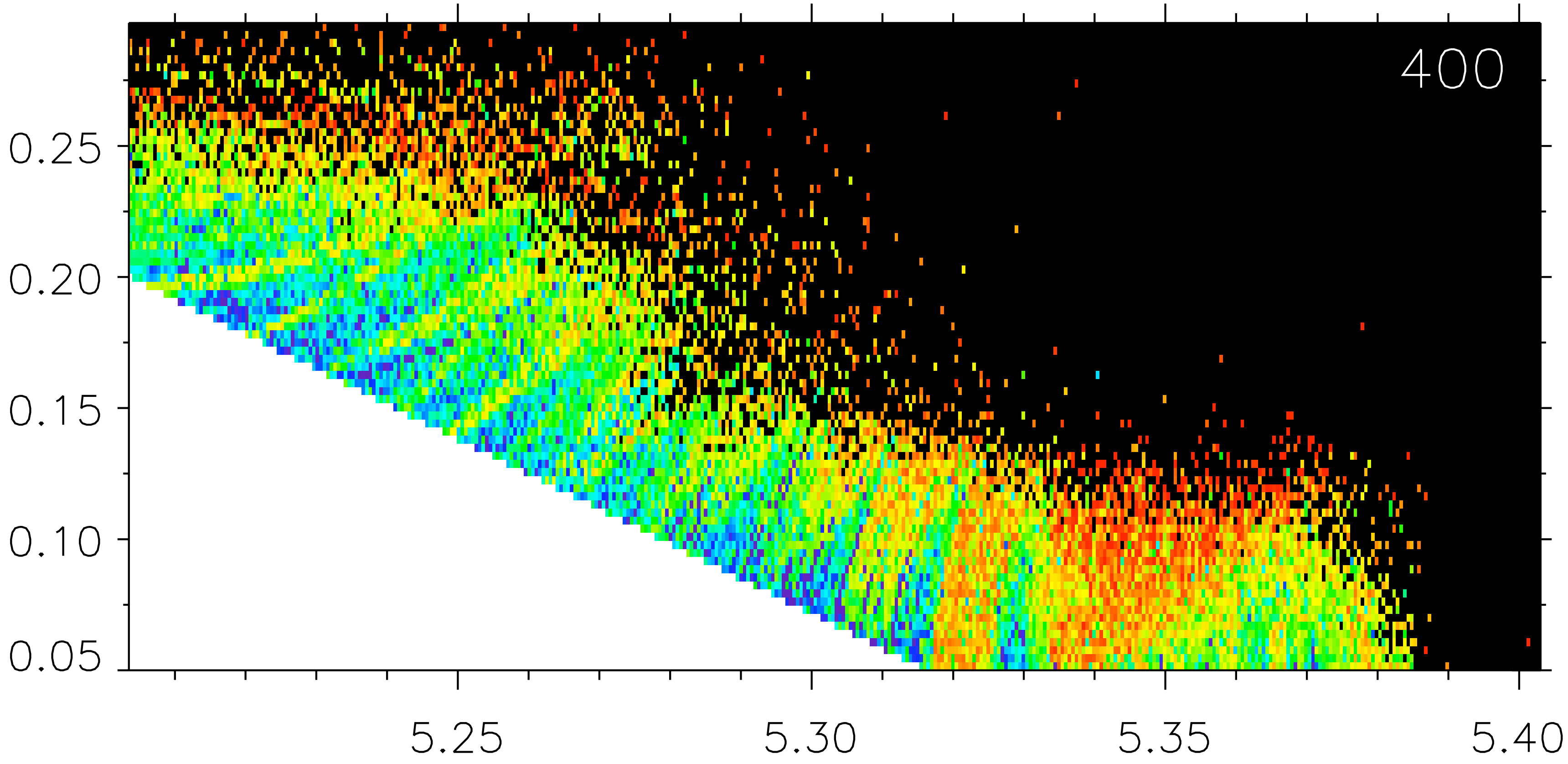
0.20
0.15
0.10
0.05
5.25
5.30
5.35
5.40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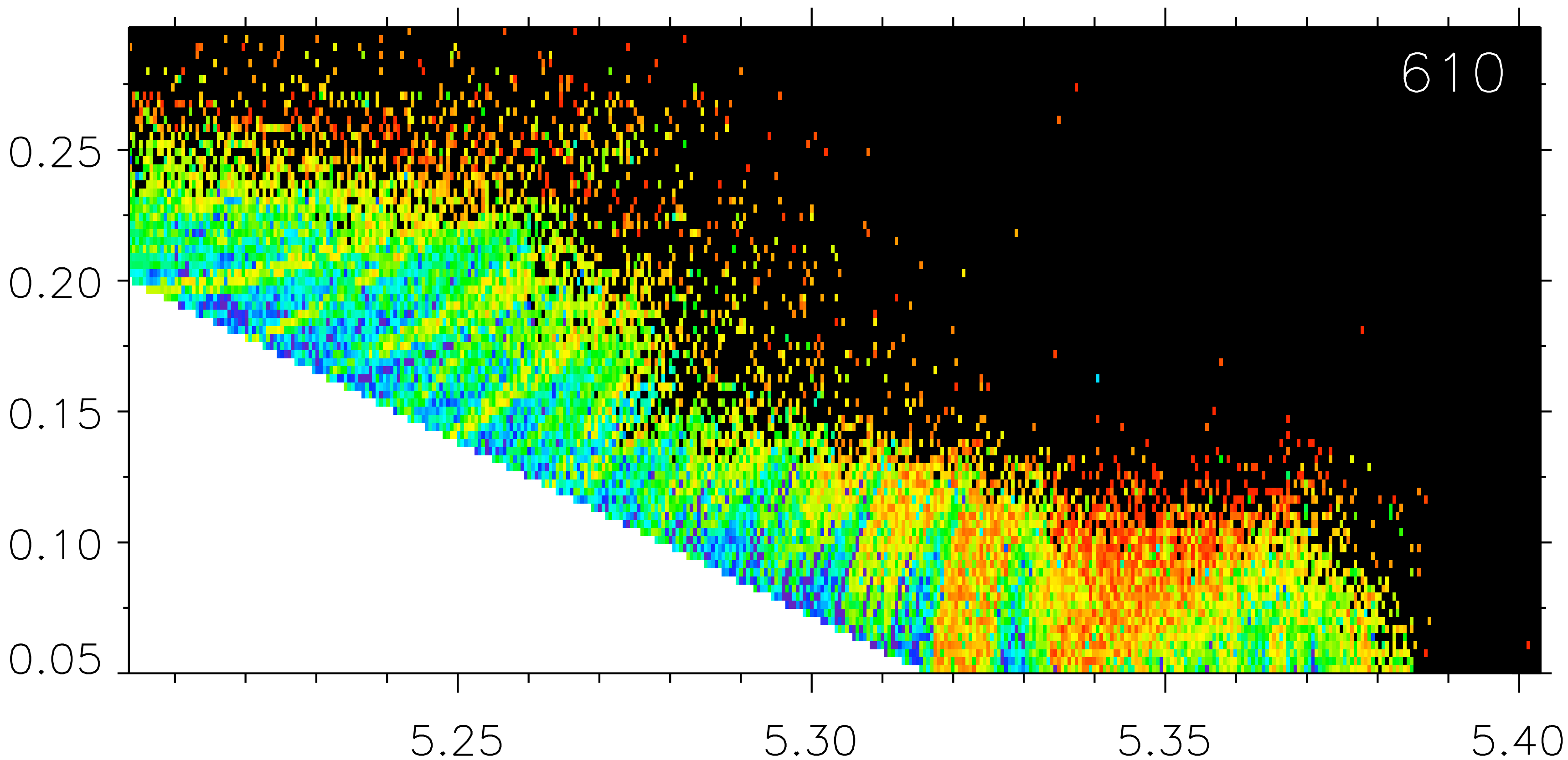
0.15
0.10
0.05
5.25
5.30
5.35
5.40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0.25
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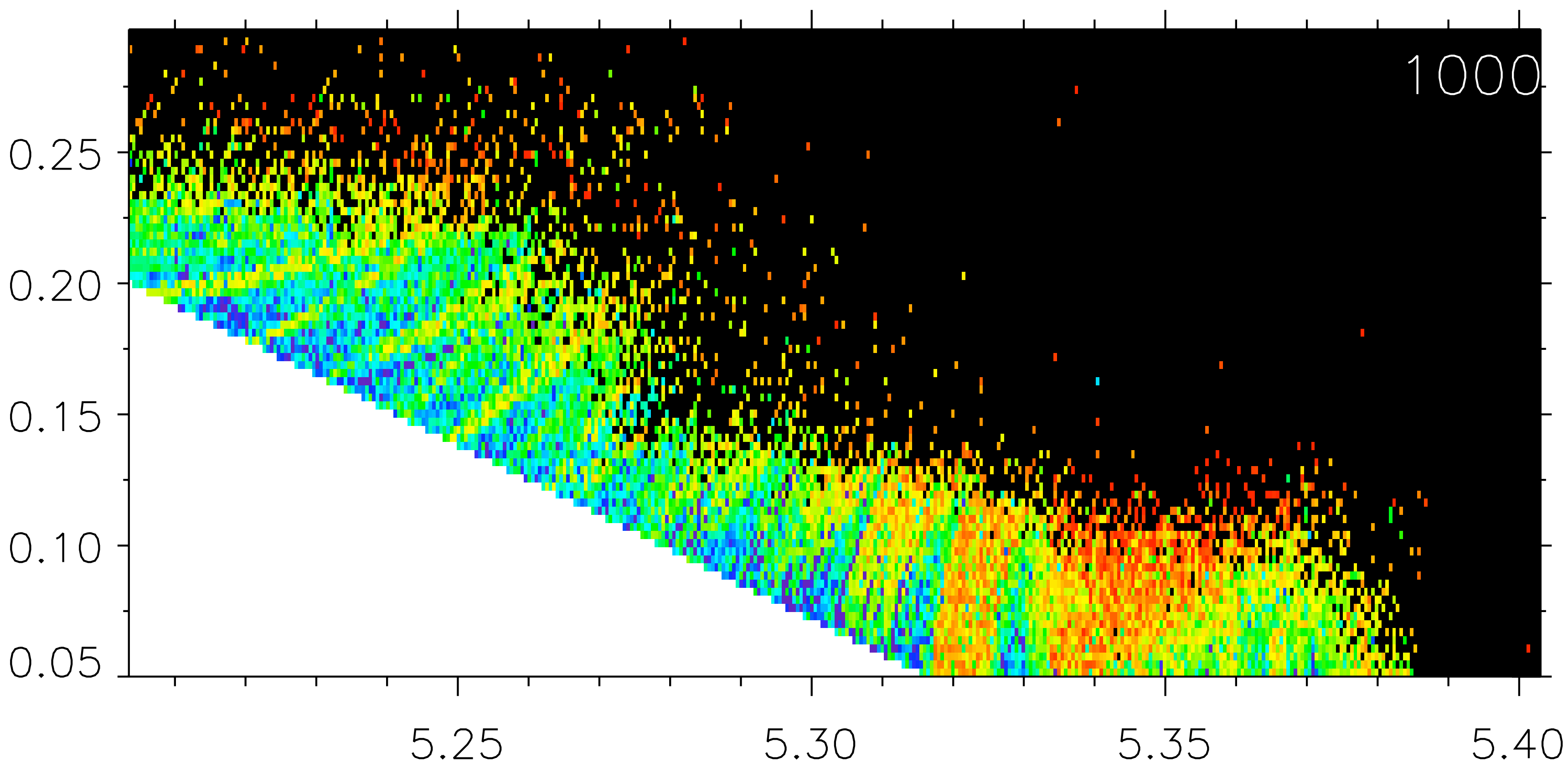
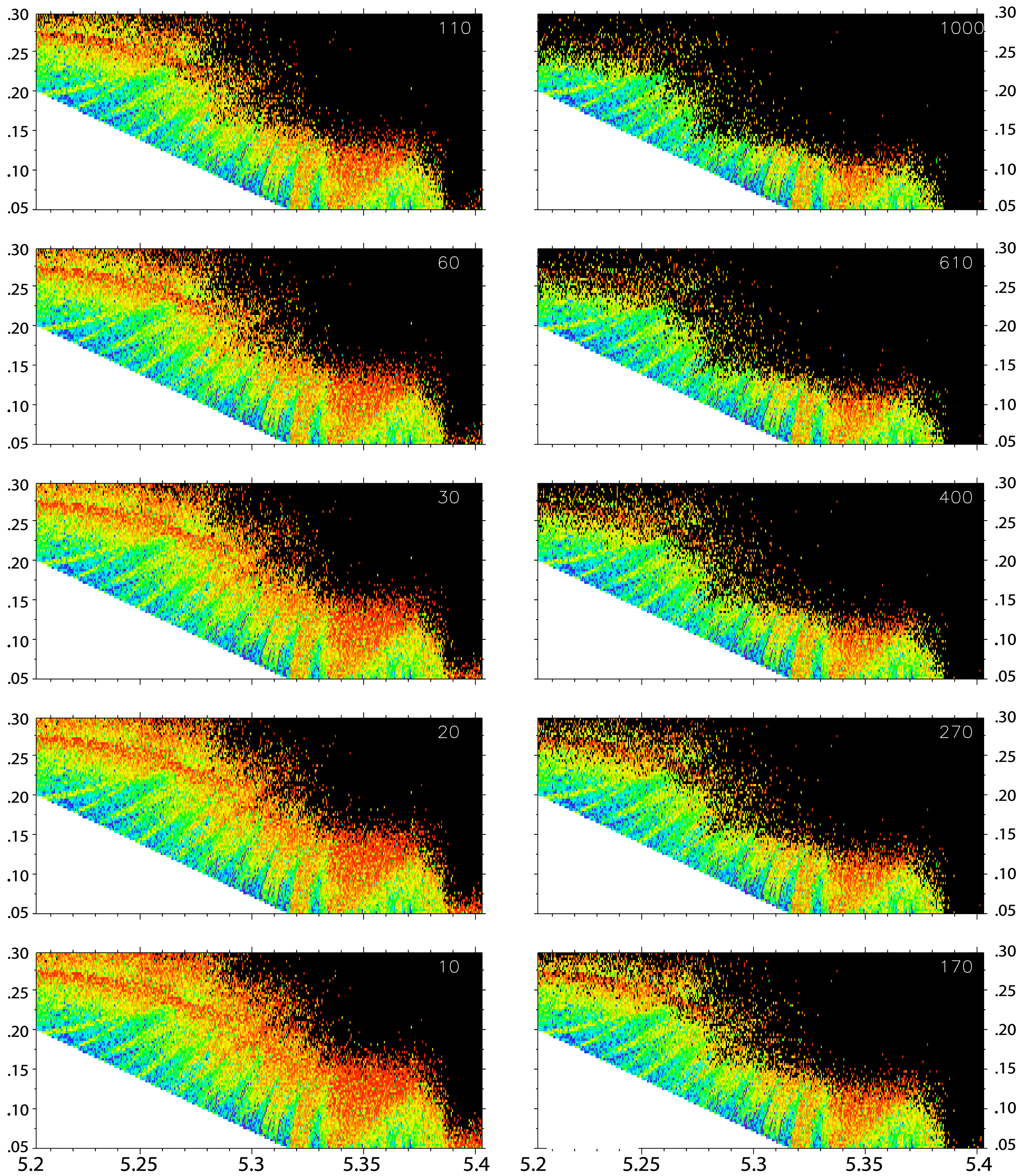
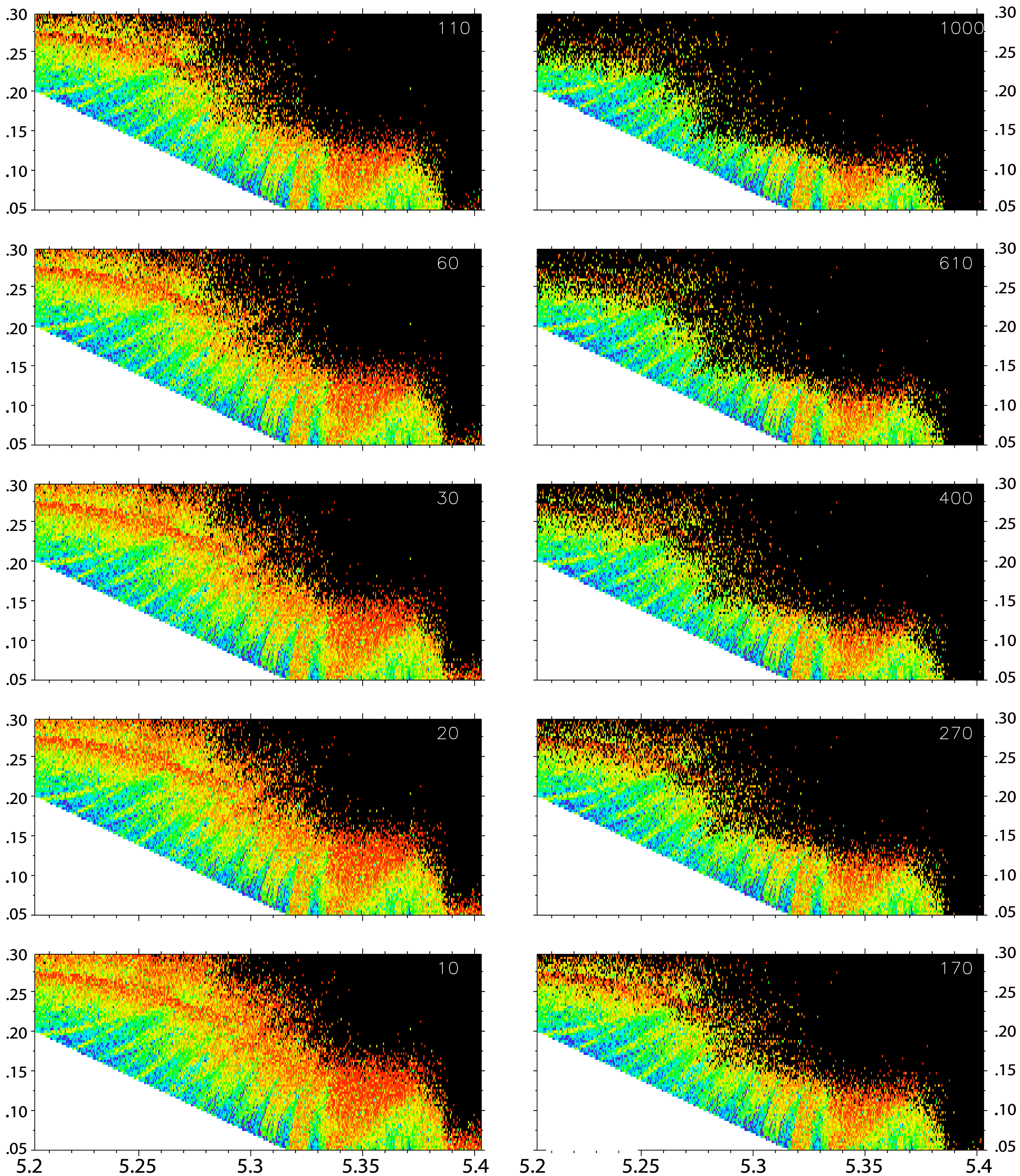
0.15
0.10
0.05
5.25
5.30
5.35
5.40


1000
0.25
0.20
0.15
0.10
0.05
5.25
5.30
5.35
5.40



Ejection in the neighborhood and above

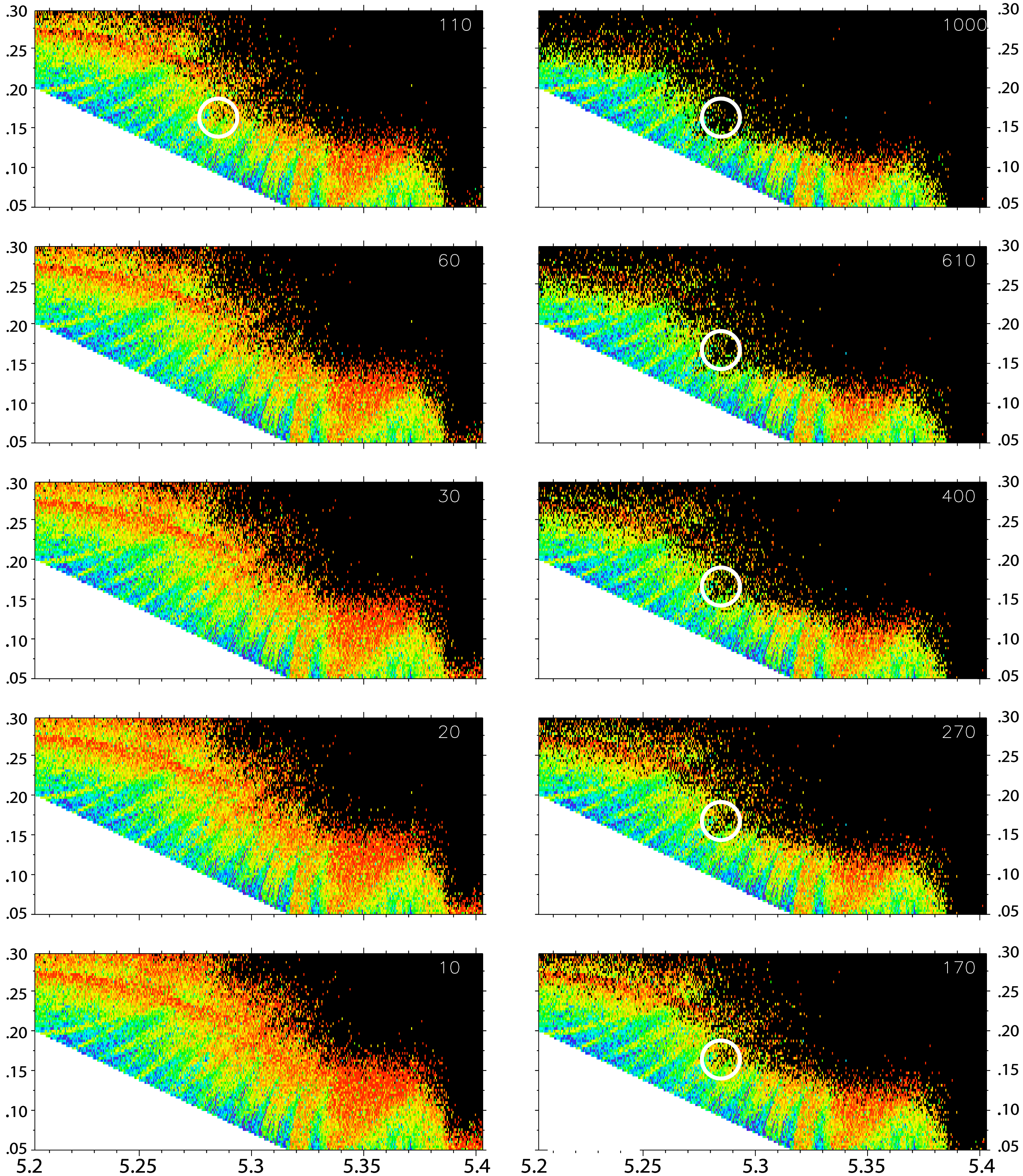
$$s = s_6$$



Ejection in the neighborhood and above

$$s = s_6$$

Regions where $\sigma > -3$ (orange, red) are cleared in 1Gy except 2



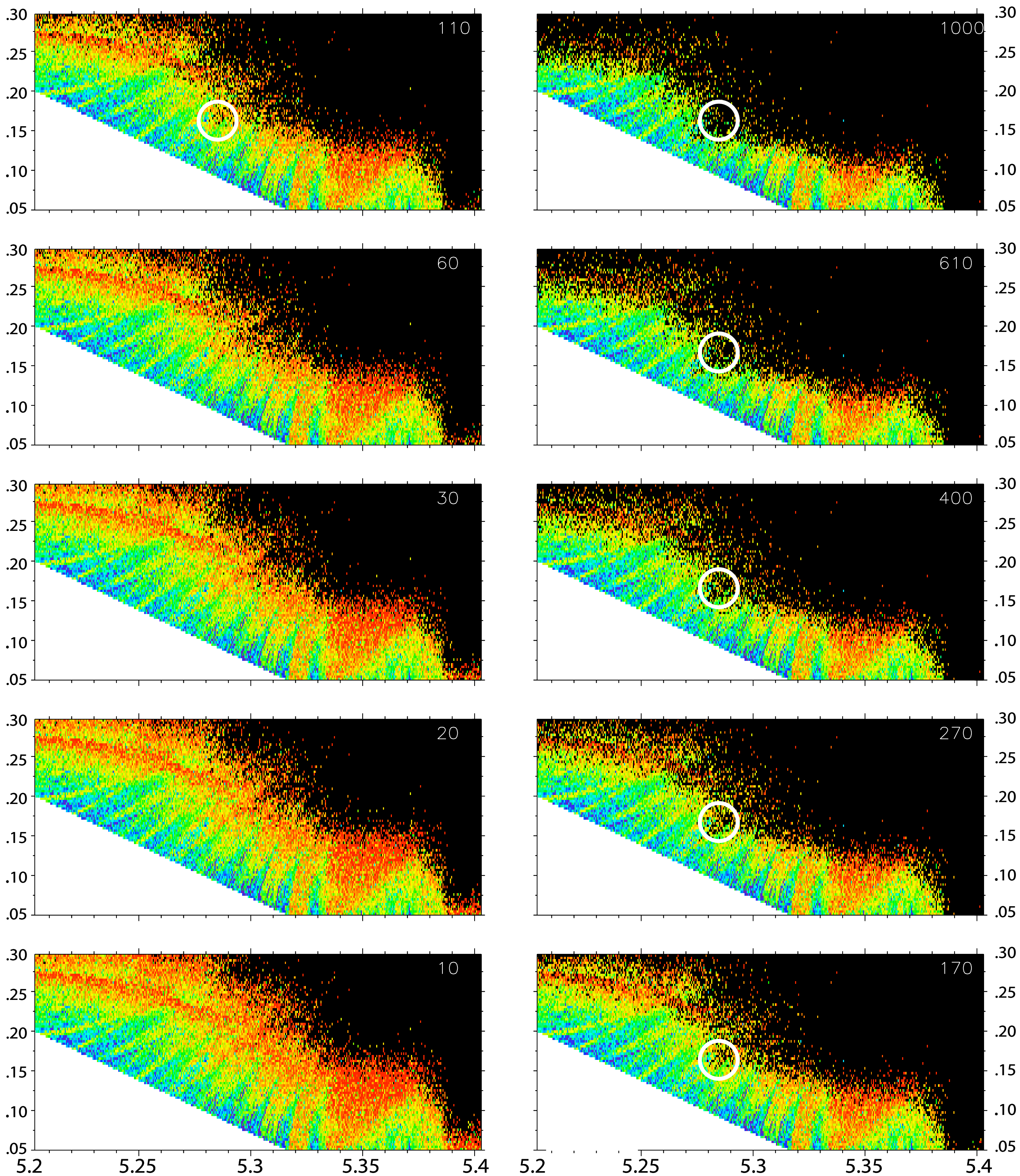
Ejection in the neighborhood and above

$$s = s_6$$

Regions where
$\sigma > -3$ (orange, red)
are cleared in 1Gy except 2

Gap along

$$4g + (2n_5 - 5n_6) - g_6 = 0$$



Ejection in the neighborhood and above

$$s = s_6$$

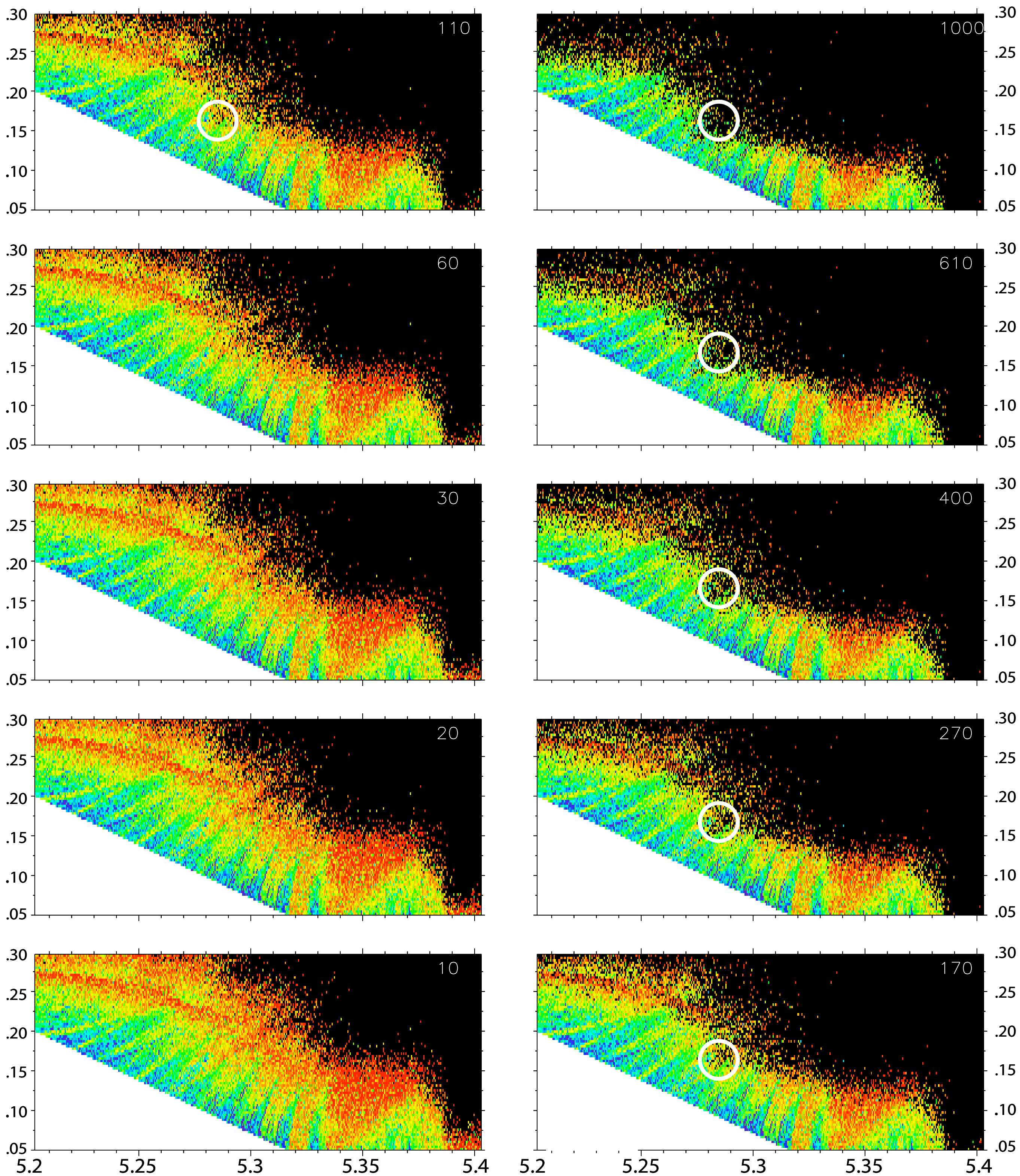
Regions where

$\sigma > -3$ (orange, red)

are cleared in 1Gy except 2

Gap along

$$4g + (2n_5 - 5n_6) - g_6 = 0$$



Ejection in the neighborhood and above

$$s = s_6$$

Regions where
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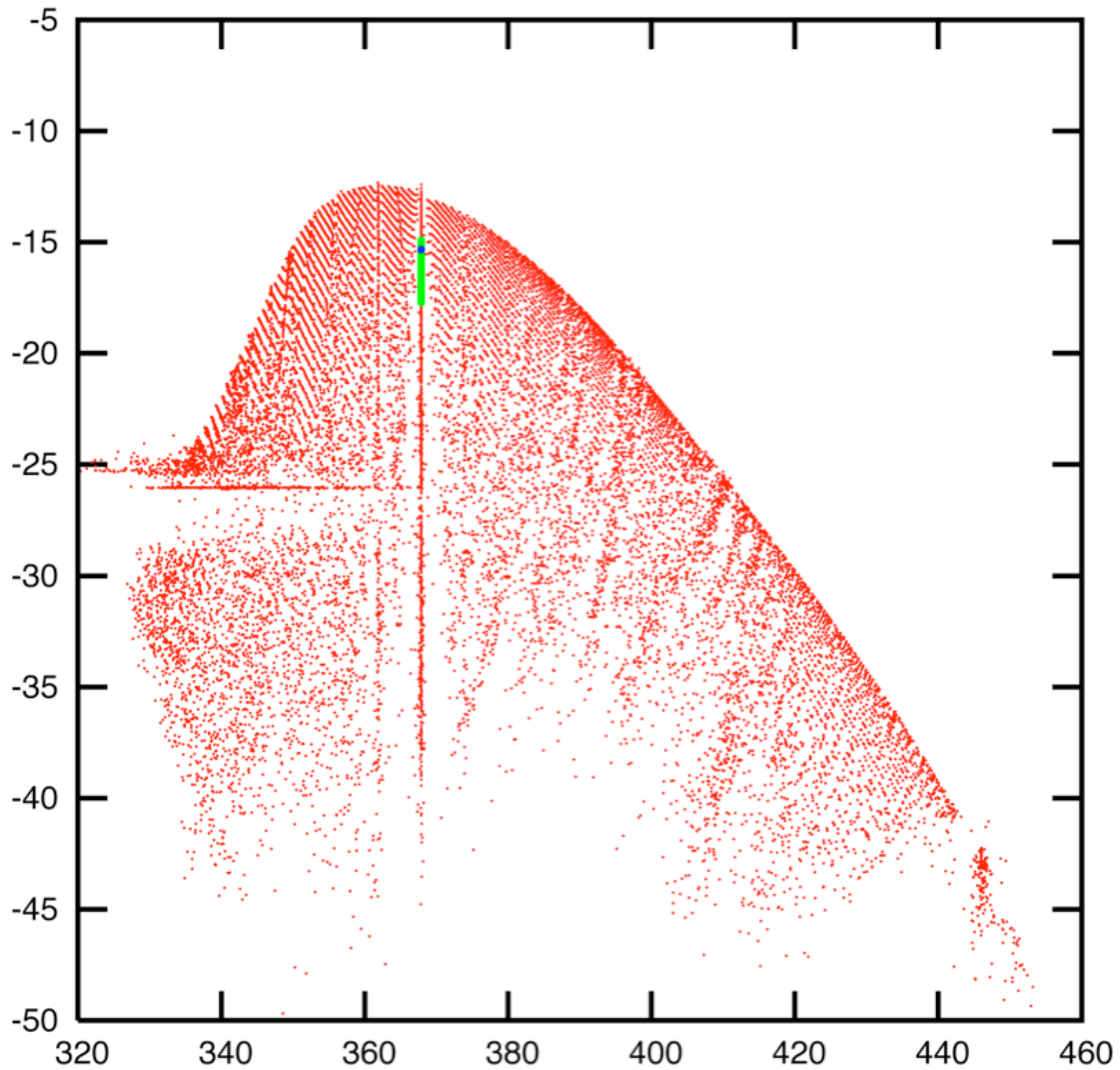
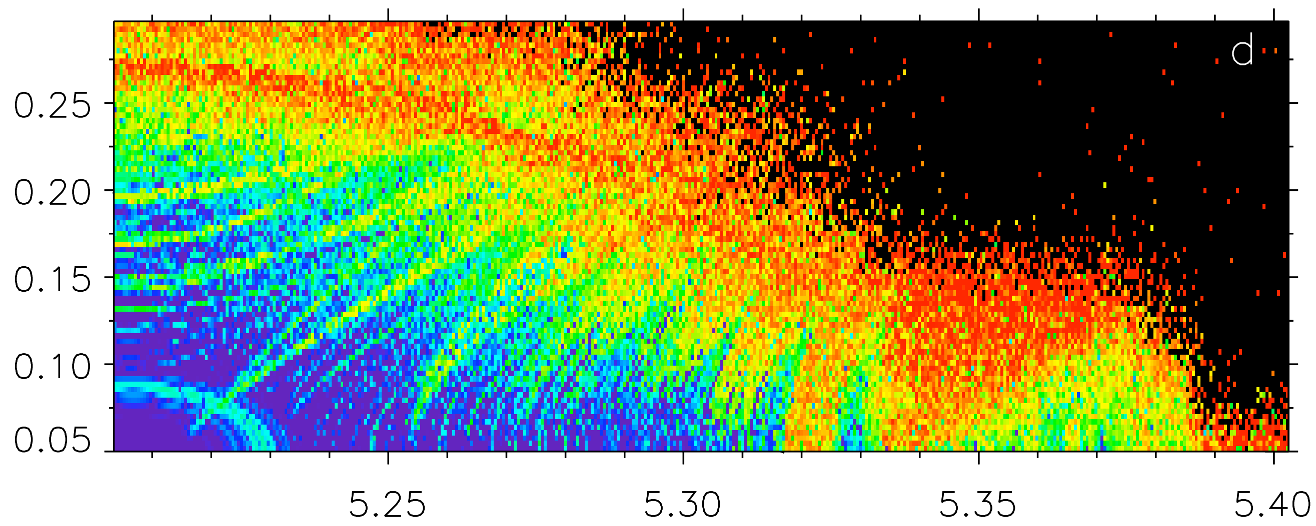
are cleared in 1Gy except 2

Gap along

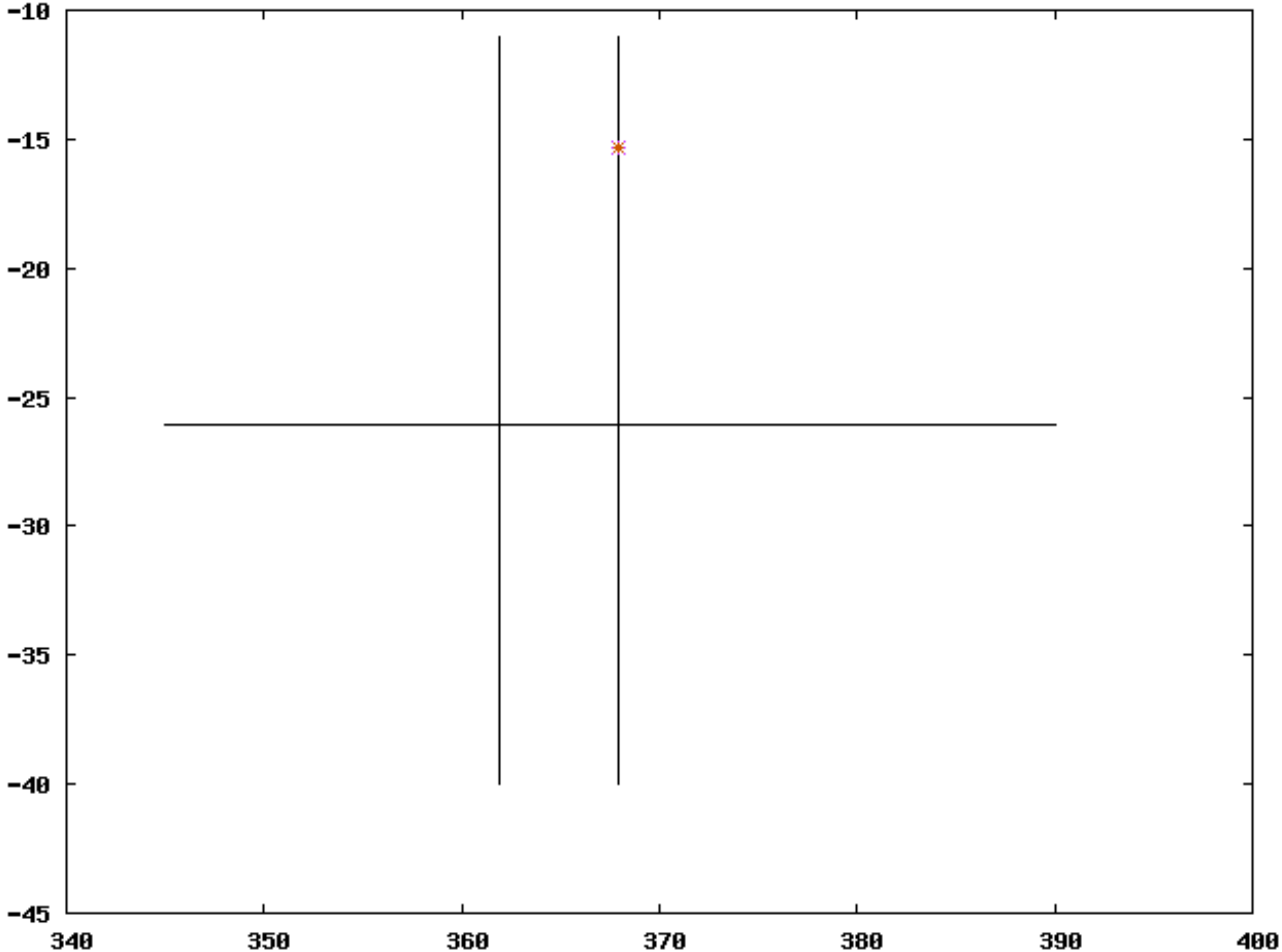
$$4g + (2n_5 - 5n_6) - g_6 = 0$$

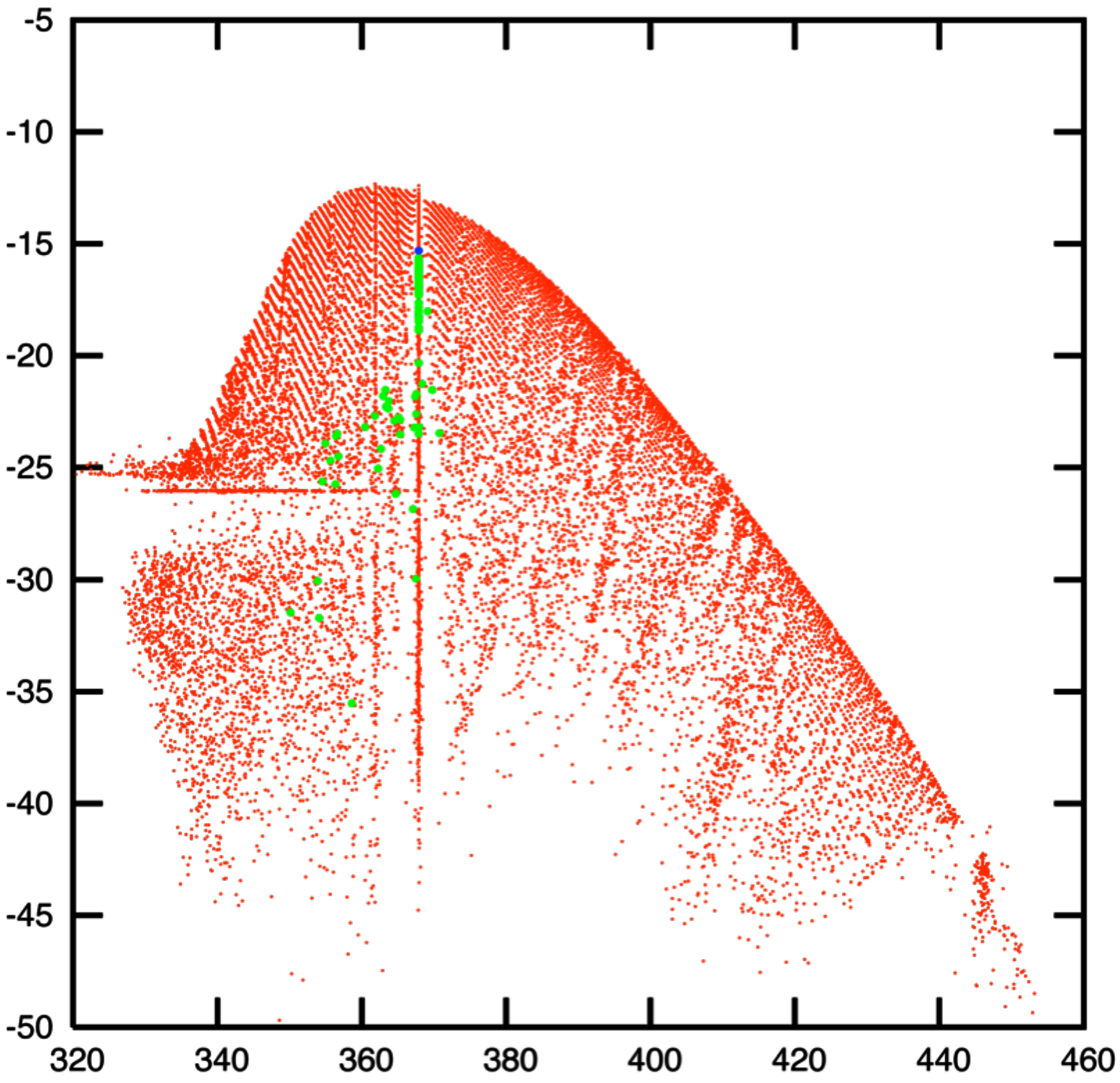




Slow diffusion along

$$4g + (2n_5 - 5n_6) - g_5 = 0$$


-10
-15
-20
-25
-30
-35
-40
-45
340
350
360
370
380
390
400



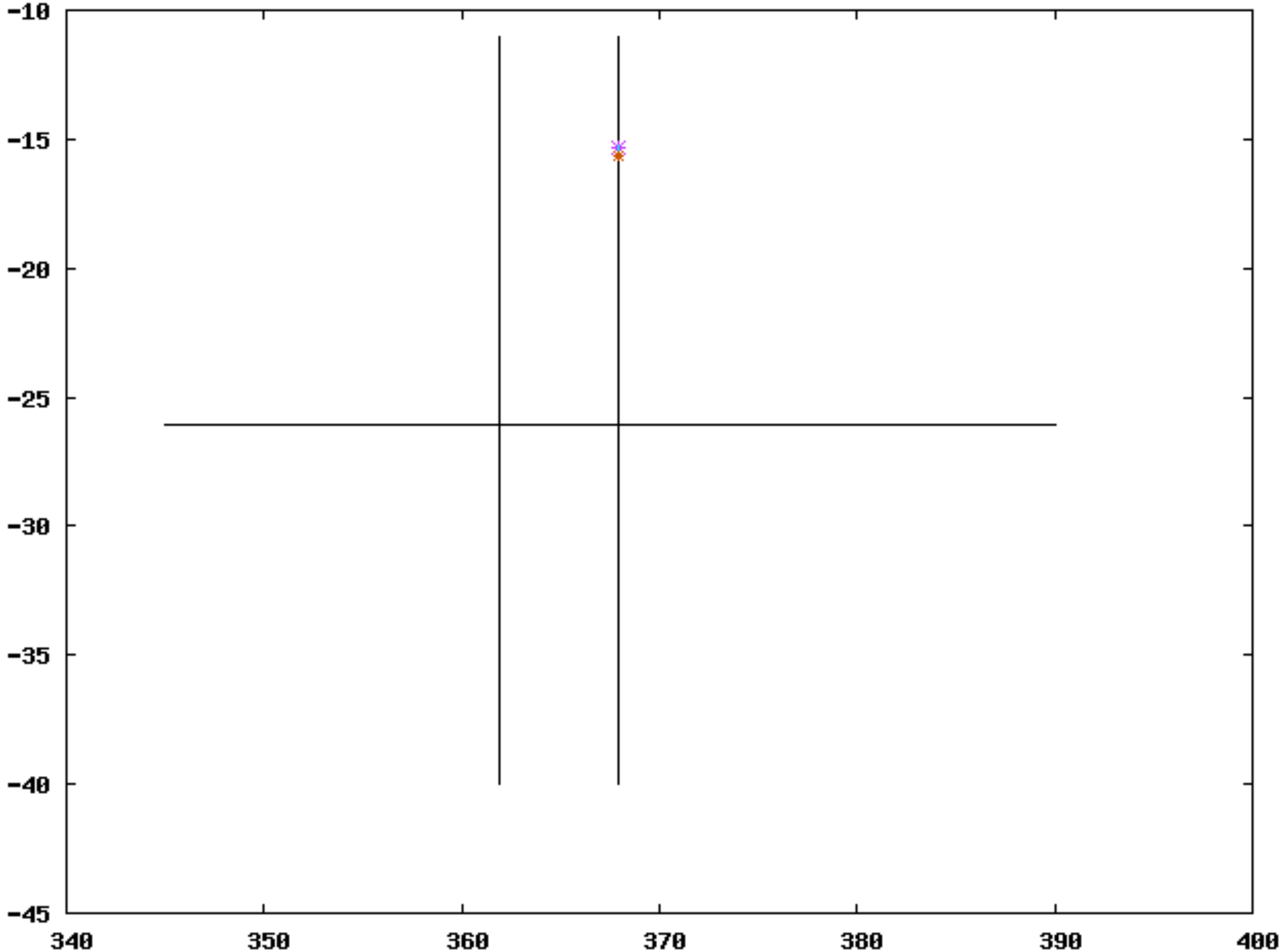
Slow diffusion along

$$4g + (2n_5 - 5n_6) - g_5 = 0$$

during 600 My

then wandering for 200 Ma

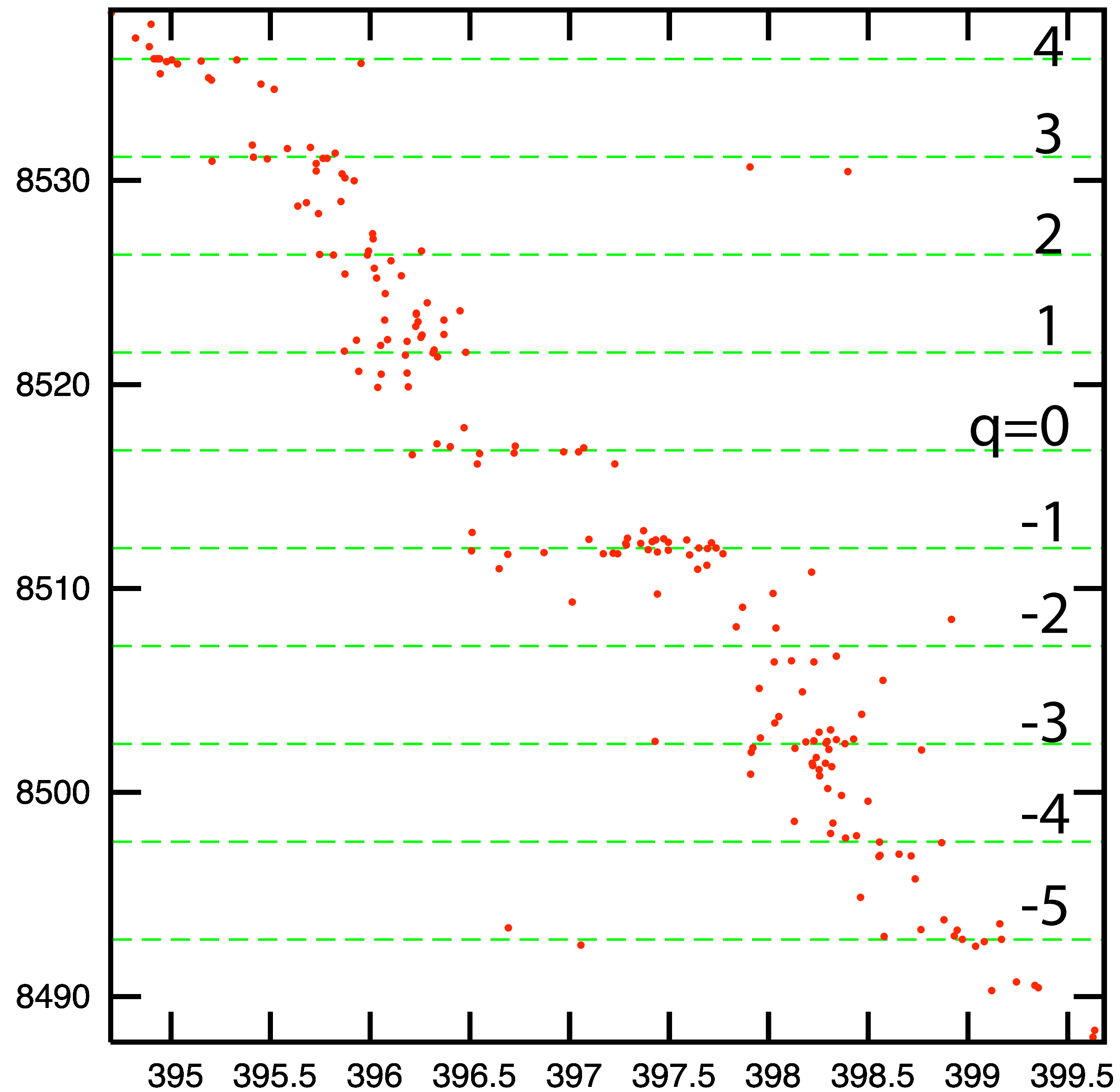
and ejection at 800 My

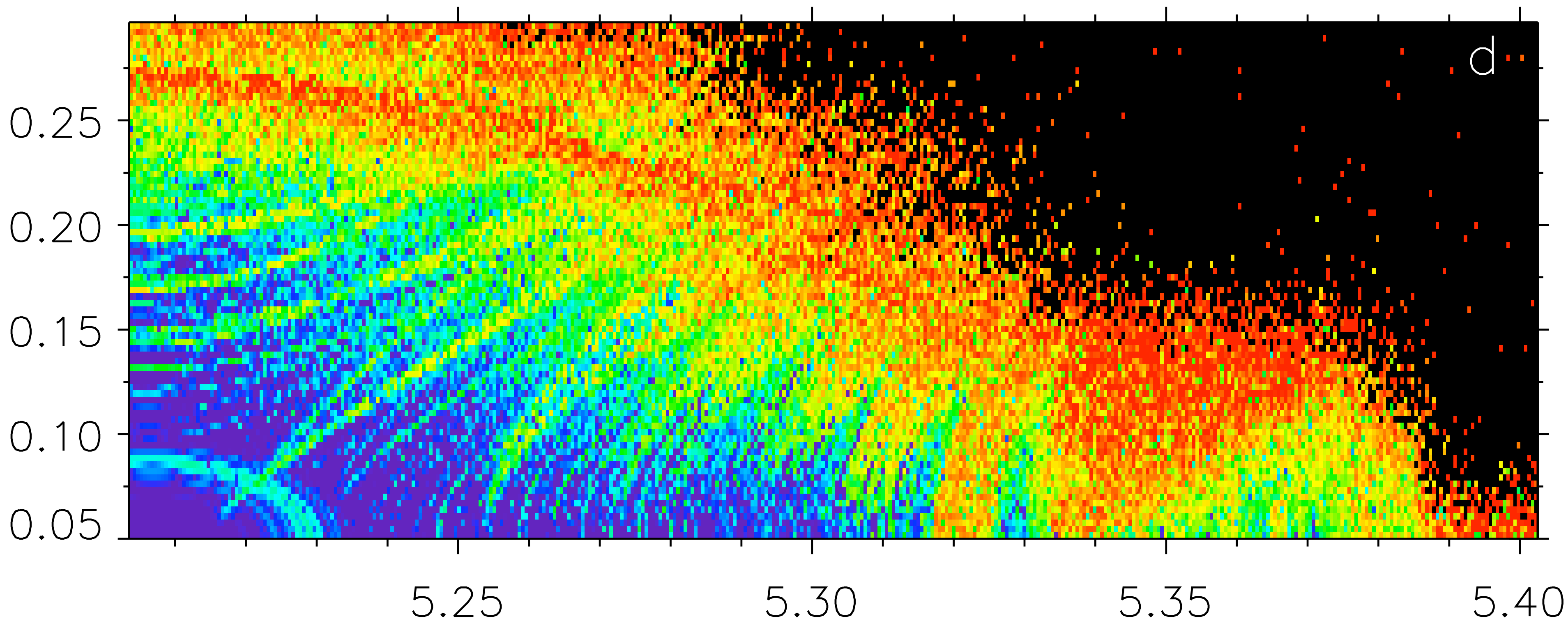

-10
-15
-20
-25
-30
-35
-40
-45
340
350
360
370
380
390
400

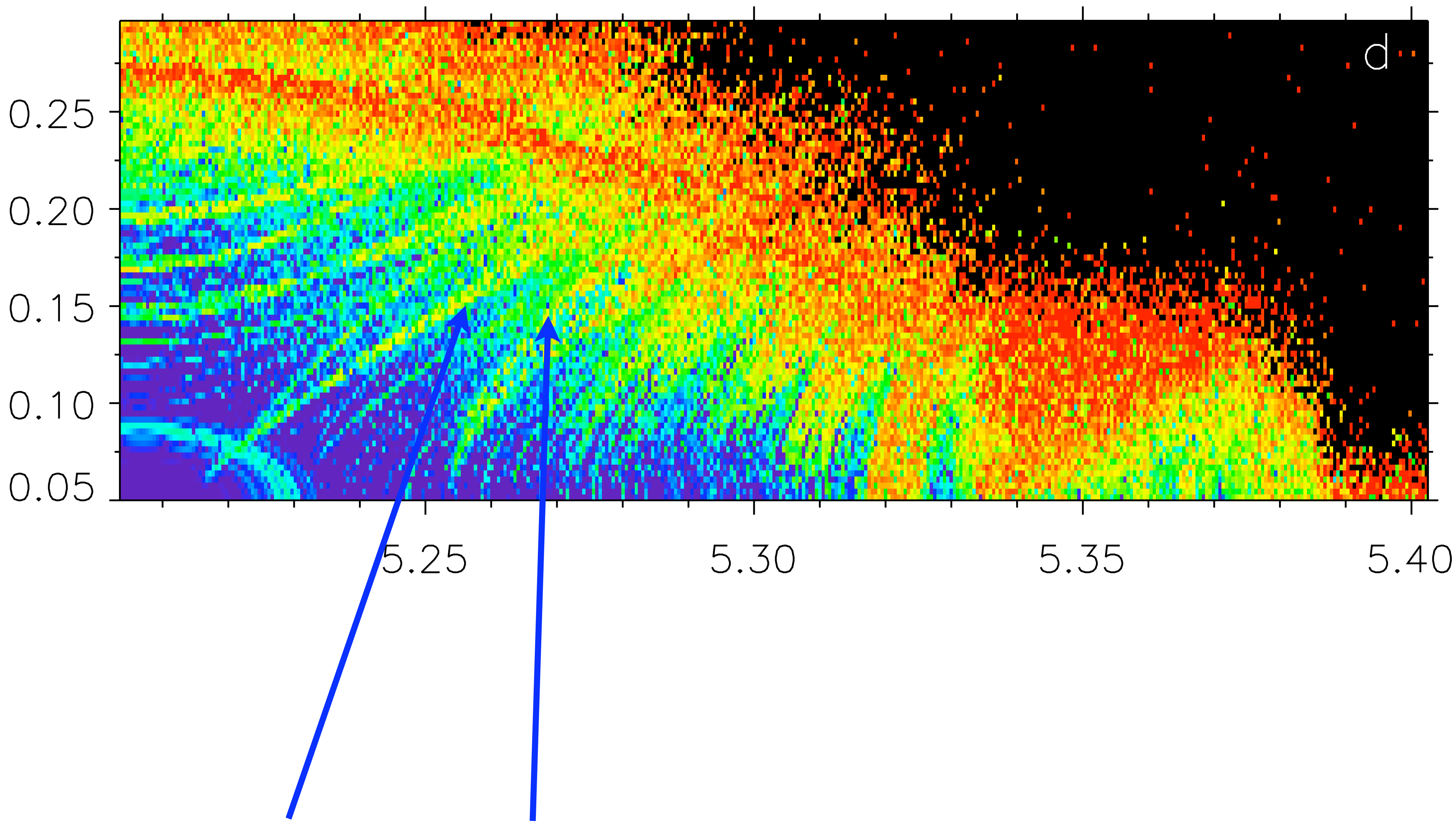
Overlapping in family II

$$5\nu - 2(n_5 - 2n_6) - 0g + qg_5 - (q+2)g_6 = 0$$

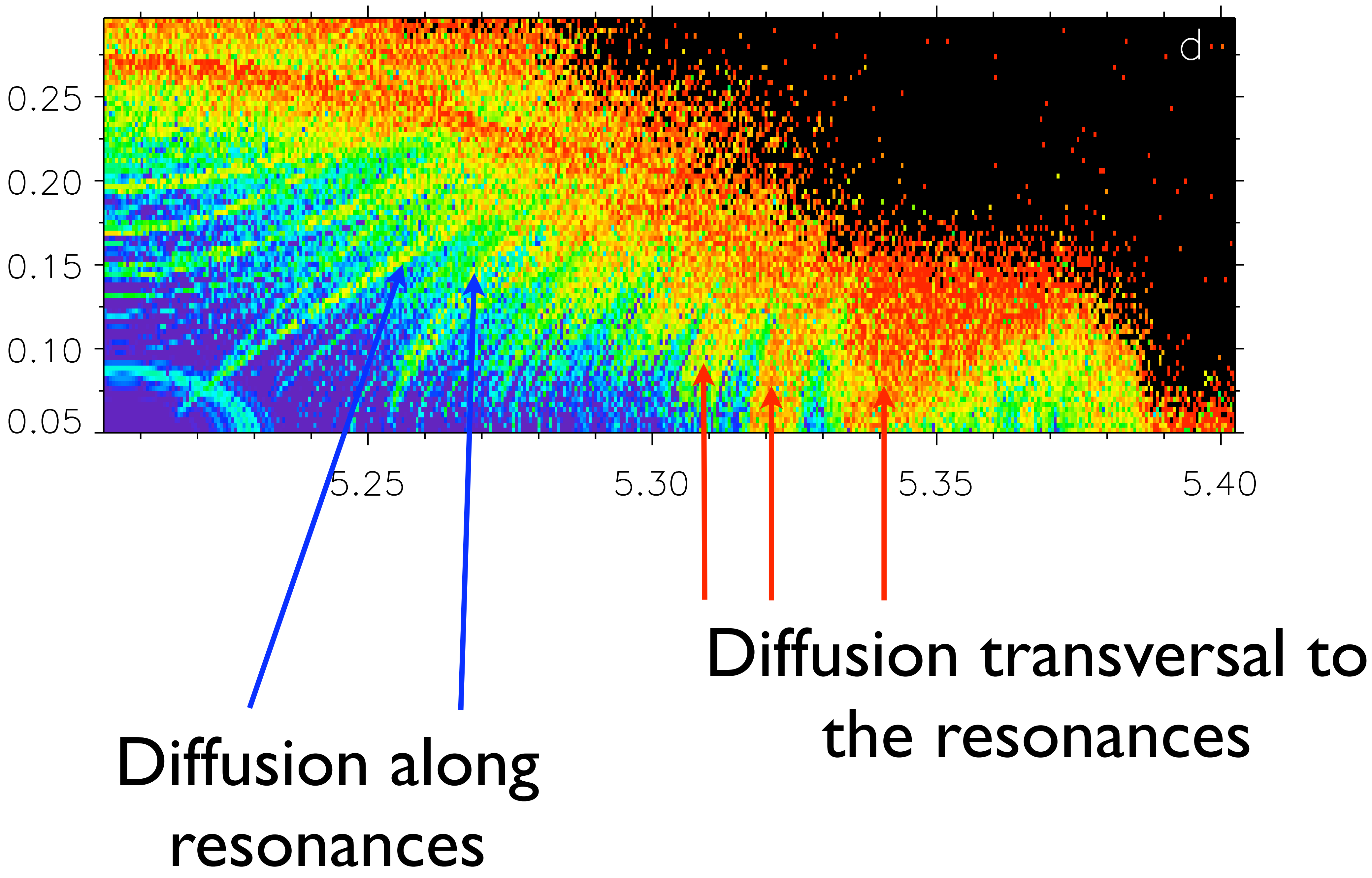
bounded diffusion




d
0.25
0.20
0.15
0.10
0.05
5.25
5.30
5.35
5.40



Diffusion along
resonances


d
0.25
0.20
0.15
0.10
0.05
5.25
5.30
5.35
5.40
Diffusion along resonances
Diffusion transversal to the resonances