FORMATION FLIGHT OF SATELLITES AROUND THE EARTH

(CELESTIAL MECHANICS SEMINAR)

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DOCTORAL PROGRAMME IN AEROSPACE SCIENCE AND TECHNOLOGY

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CONCEPTS

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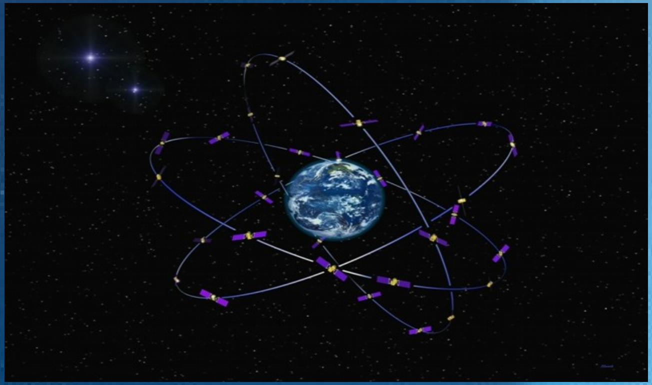
Formation Versus Constellation

*

As defined by the NASA Goddard Space Flight Centre, a constellation is composed of two or more spacecraft in similar orbits with no active control by either to maintain their relative position. Is only necessary that spacecrafts maintain themselves within their own pre-specified boxes without collisioning or changing the overall coverage of the Earth significantly. So, the consideration of relative positions or orientations, real-time coordination and a very high level of autonomy is not required.



In contrast, a *formation is two or more spacecraft that use an active control scheme to maintain the relative positions between spacecrafts*. It has to be a direct control on the relative position and orientation between one spacecraft and another one (typically its neighbour) or many other. This means that active, real-time and closed-loop control is required.

ADVANTAGES

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Many potential applications of this enabling technology exist, one of wich is to improve the performance of Earth observation. A cluster of satellites will be able to synthesize a much larger aperture than can be achieved with a single platform, thus providing signicant increases in imagin resolution through interferometry.

The cluster approach has many advantages over a single large satellite.

- Each spacecraft is smaller, lighter and simpler to manufacture;
  for these reasons a cluster of many satellites is les expensive
  and less complex than a single large satellite.
- The failure of a component of a single satellite is not critical in
  a cluster because failed satellites can be increment replaced
  and the cluster is easily adapted to the failure.
- Lower consume of fuel used in transportation.
- Higher angular and spatial resolution imagery and interferometry
  as well as the sensitivity of scientic instruments.

FORMATION FLIGHT MISSIONS

Nowadays the list of missions that implement the formation flight configuration is quite large, taking into account that the concept of formation flight is relatively new. The following list includes some of these missions. Some of them are briefly described later:

1. EO-1 (Earth Orbiter-1)
2. GRACE (Gravity Recovery and Climate Experiment)
3. TechSat21 (Technology Satellite of the 21st Century)
4. XEUS (X-Ray Evolving Universe Spectometrer)
5. MAXIM (Micro-Arcsecond X-ray Imaging Mission)
6. LISA (Laser Interferometer Space Antenna)
7. PRISMA
8. PROBA-3
9. Constellation-X
10. TPF (Terrestrial Planet Finder)
11. Planet Imager
12. DS3 (Deep Space 3)
13. DARWIN
14. SIMBOL X
15. FIMOS (Fire Monitoring Satellite Swarm)
16. ION-F (Ionospheric Observation Nanosatellite Formation)
17. SMARTSAT-1
18. MAGNAS (Magnetic NanoProbe Swarm)
19. Gemini (GPS-based Orbit Estimation and Laser Metrology for Intersatellite Navigation)
20. AMSAT

FORMATION FLIGHT – MISSIONS I

Darwin and TPF

Darwin (ESA) and Terrestrial Planet Finder (NASA) are two formation projects which were planned to find planets with the ability to support life.
The goal of the projects is to measure the size, temperature, and placement of planets similar to Earth in the habitable zones of distant solar systems.
For this purpose, the two projects are based on a formation of spacecraft with a virtual aperture of thousands of meters

Terrestrial Planet Finder (TPF)

The Terrestrial Planet Finder concept sets a large baseline of a hundred of meters, composed by four telescope spacecraft with a diameter of 3 or 4 meters. These four spacecraft are combined with a collector, in an equilateral triangle with the two inner spacecraft of the baseline.

The five spacecraft form a rotating formation, and there are two target orbits for the formation: one is a Halo orbit about the L2 Sun - Earth+Moon system, and the other one is a heliocentric orbit similar to the SIRTF orbit

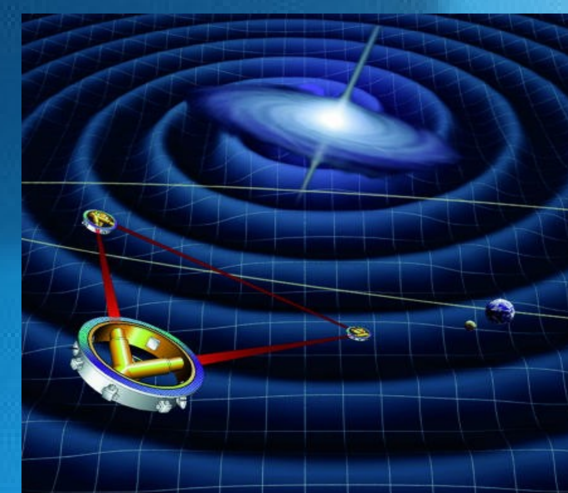
FORMATION FLIGHT – MISSIONS II

ST5 (Space Technology 5)

Objective: To test and validate new technologies and aid scientists in understanding the harsh environment of the Earth's magnetosphere. Using data collected from the ST5 constellation, scientists can begin to understand and map the intensity and direction of the magnetic field, its relation to space weather events, and the affects on our planet. Is part of NASA’s New Millenium Program (NMP)

LISA

The Laser Interferometer Space Antenna (LISA) is a joint project of ESA and NASA to study the mergers of super massive black holes, tests Einstein’s Theory of General Relativity, probes the early Universe, and searches for gravitational waves the primary objective. The LISA formation is a large formation. It has three spacecraft in the vertices of an equilateral triangle, with a distance of five million kilometers between each spacecraft



The plane of the triangle formed by the spacecraft is inclined at an angle of 60° to the plane of the ecliptic. This position is chosen to minimize the gravitational disturbances from the Earth-Moon system and to admit the communication with Earth

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RECENT ADVANCES IN FORMATION FLIGHT

***Sedwick, Kong and Miller [1] used the Clohessy-Wilshire equations as their starting model to find relative orbits about a reference spacecraft*. A circular reference orbit and a spherical Earth (without J2 effect) was assumed in their study. They used these linear equations of motion to establish a large family of relative orbits.**

/ *Schaub and Alfriend [2] built on the work of Brouwer [3] and found J2 invariant relative orbits. Working with mean orbit elements*, the secular drift of the longitude of the ascending node and the sum of the argument of perigee and mean anomaly were set equal between two neighboring orbits. By having both orbits drift at equal rates on the average, they would not pull apart over time due to the J2 influence.

***Dong-Woo Gim and Kyle T. Alfriend [4] used a precise analytic solution for the relative motion of satellites. Based on the relationship between the relative states and the differential orbital elements,* the state transition matrix for the linearized relative motion that includes the effects due to the reference orbit eccentricity and the gravitational perturbations is derived. This method is called the Geometric Method.**

***In last years, Sabatini M, D. Izzo and R. Bevilacqua [5] see the possibility of obtaining a natural periodic relative motion of formation flying Earth satellites (numerically and analytically)*. The numerical algorithm is based on a genetic strategy. First, They tested their algorithm using a point mass gravitational model. In this case the period matching between the considered orbits is a necessary and sufcient condition to obtain invariant relative trajectories. Then, the J2 perturbed case is considered.**

RECENT ADVANCES IN FORMATION FLIGHT - METHODS

Clohessy-Wiltshire (CW) Equations [8]

Use RSW coordinate system *(different from NASA)*

Target satellite has two-body motion:

$$\ddot{\bar{r}}_{tgt} = -\frac{\mu\,\bar{r}_{tgt}}{r_{tgt}^3}$$

The interceptor is allowed to have thrusting

$$\ddot{\bar{r}}_{int} = -\frac{\mu\,\bar{r}_{int}}{r_{int}^3} + \bar{F}$$

Then $\bar{r}_{rel} = \bar{r}_{int} - \bar{r}_{tgt} \Rightarrow \ddot{\bar{r}}_{rel} = \ddot{\bar{r}}_{int} - \ddot{\bar{r}}_{tgt}$

So, $\ddot{\bar{r}}_{rel} = -\frac{\mu\,\bar{r}_{int}}{r_{int}^3} + \bar{F} + \frac{\mu\,\bar{r}_{tgt}}{r_{tgt}^3}$

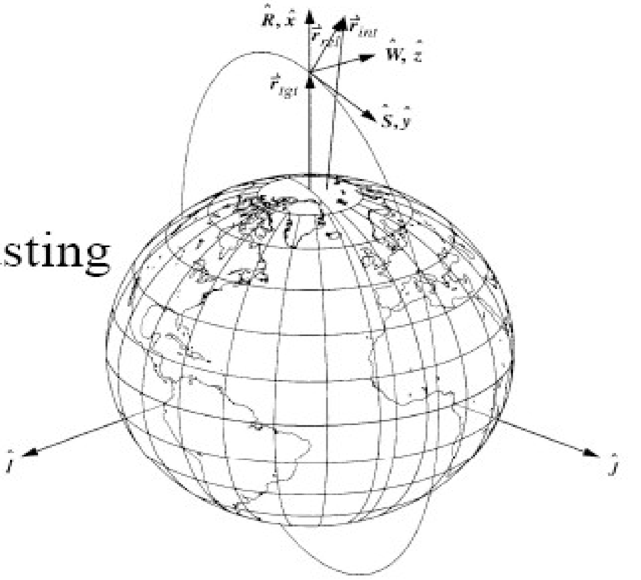


Figure 5-25. Coordinate-System Geometry for Relative Motion. The *xyz* notation reminds us the results are approximate. Remember that the *S* (and *y*) axes will be aligned with the velocity vector only in circular orbits. A major assumption in relative motion is that the target satellite is in a nearly circular orbit.

$$\begin{aligned} \ddot{x} - 2\omega\dot{y} - 3\omega^2 x &= f_x \\ \ddot{y} + 2\omega\,\dot{x} &= f_y \\ \ddot{z} + \omega^2 z &= f_z \end{aligned}$$

RECENT ADVANCES IN FORMATION FLIGHT - METHODS

Clohessy-Wiltshire (CW) Equations [1]

The above equations can be solved (see book) leaving:

$$x(t) = \frac{\dot{x}_0}{\omega}\sin\omega t - \left(3x_0 + \frac{2\dot{y}_0}{\omega}\right)\cos\omega t + \left(4x_0 + \frac{2\dot{y}_0}{\omega}\right)$$

$$y(t) = \left(6x_0 + \frac{4\dot{y}_0}{\omega}\right)\sin\omega t + \frac{2\dot{x}_0}{\omega}\cos\omega t - (6\omega x_0 + 3\dot{y}_0)t + \left(y_0 - \frac{2\dot{x}_0}{\omega}\right)$$

$$z(t) = z_0\cos\omega t + \frac{\dot{z}_0}{\omega}\sin\omega t$$

$$\dot{x}(t) = \dot{x}_0\cos\omega t + (3\omega x_0 + 2\dot{y}_0)\sin\omega t$$

$$\dot{y}(t) = (6\omega x_0 + 4\dot{y}_0)\cos\omega t - 2\dot{x}_0\sin\omega t - (6\omega x_0 + 3\dot{y}_0)$$

$$\dot{z}(t) = -z_0\omega\sin\omega t + \dot{z}_0\cos\omega t$$

So, given $x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0$ of interceptor, can compute $x, y, z,$ $\dot{x}, \dot{y}, \dot{z}$ of interceptor at future time.

We can also determine ΔV needed for rendezvous. Given x_0, y_0, z_0, we want to determine $\dot{x}_0, \dot{y}_0, \dot{z}_0$ necessary to make x=y=z=0. Set first 3 equations to zero, and solve for $\dot{x}_0, \dot{y}_0, \dot{z}_0$.

$$\dot{x}_0 = -\frac{\omega x_0(4 - 3\cos\omega t) + 2(1 - \cos\omega t)\dot{y}_0}{\sin\omega t}$$

$$\dot{y}_0 = \frac{(6x_0(\omega t - \sin\omega t) - y_0)\omega\sin\omega t - 2\omega x_0(4 - 3\cos\omega t)(1 - \cos\omega t)}{(4\sin\omega t - 3\omega t)\sin\omega t + 4(1 - \cos\omega t)^2}$$

$$\dot{z}_0 = -z_0\omega\cot\omega t$$

Assumptions:

1. Satellites only a few km apart
2. Target in circular orbit
3. No external forces (drag, etc.)

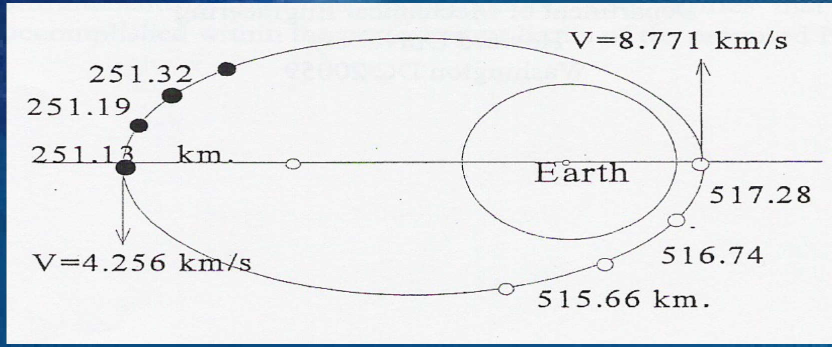
RECENT ADVANCES IN FORMATION FLIGHT - METHODS

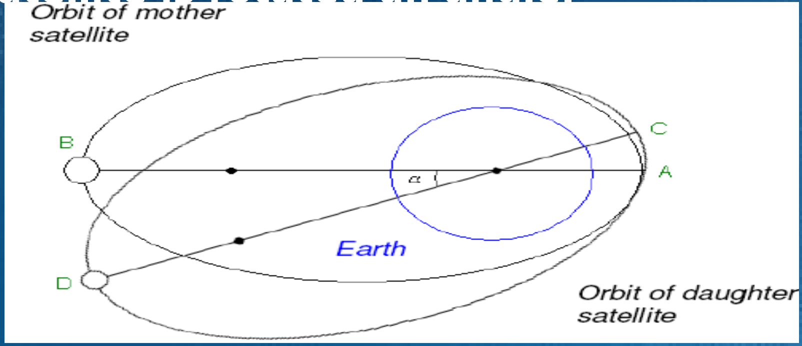
Formation flight for elliptical orbit - Peter M. Bainum [6]

The aim of this work consists on maintaining constant the separation distance between the satellites. This is trivial in a circular orbit, but in an elliptical orbit it is not as this because the inter-satellite distance is not constant, due to the differences in velocity along it. *If two satellites are in the same orbit and a nominal distance d is fixed between them, with a simple simulation can be demonstrated that the distance can vary about a 25% depending on the chosen orbit*.

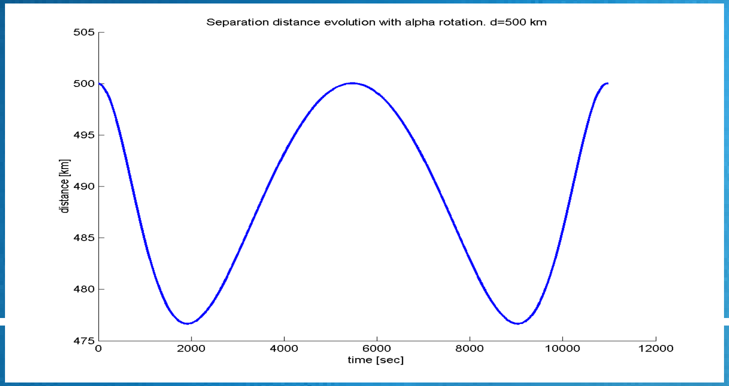
For example, In this scheme, two satellites are orbiting around the Earth in the same orbit. The perigee is at an altitude of 600 km and the apogee is at 8000 km. Both satellites are pretended to be separated 500 km during the entire orbit, but this distance does not remain constant.

The mother and daughter satellites in two equal elliptical orbits but one displaced respect to the other. The orbit of daughter satellite is rotated an angle.





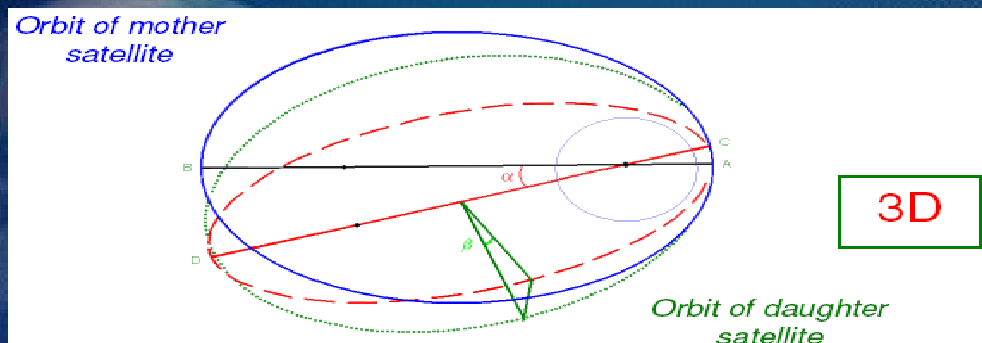
$$\alpha = \frac{d}{2a}$$



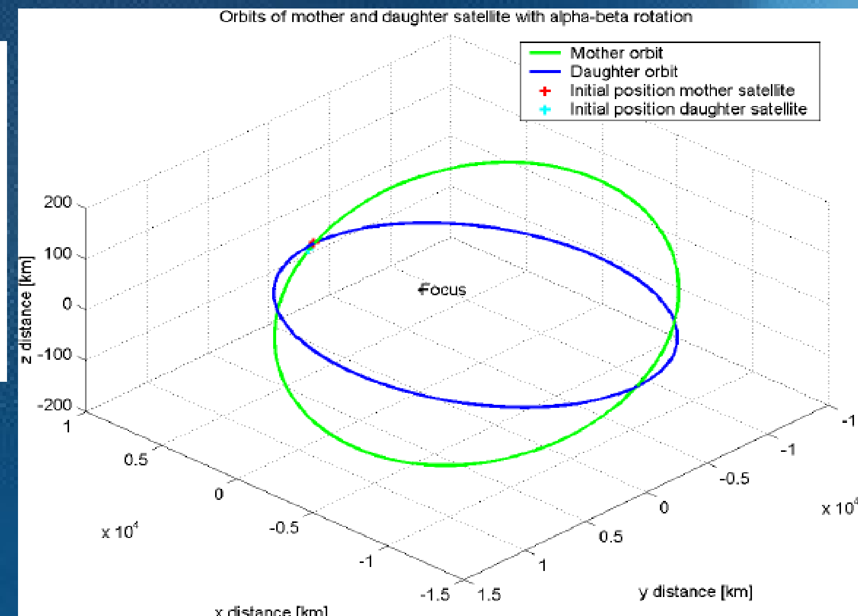
RECENT ADVANCES IN FORMATION FLIGHT - METHODS

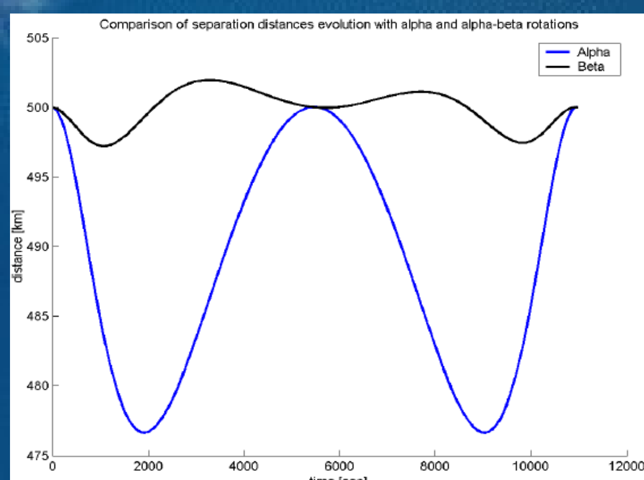
Elliptical orbit in formation flight - Peter M. Bainum [6]

***It is possible to reduce more the separation distance between satellites error by doing another rotation to the daughter's orbit*. In this case, the semi-major axis of the daughter satellite orbit will be rotated an angle beta , as is illustrated in the following figure**



3D





$$\beta = \frac{\sqrt{d^2 - s^2}}{b}$$

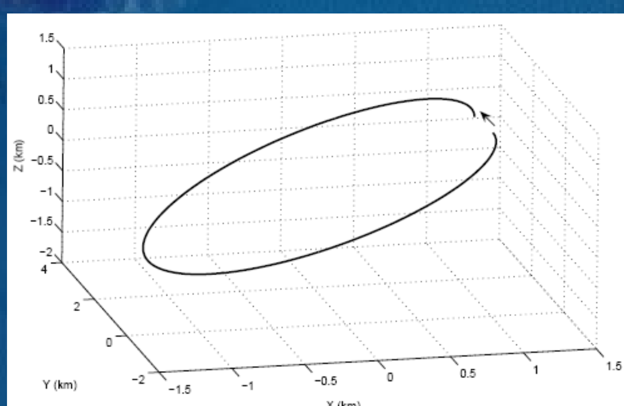
RECENT ADVANCES IN FORMATION FLIGHT - METHODS

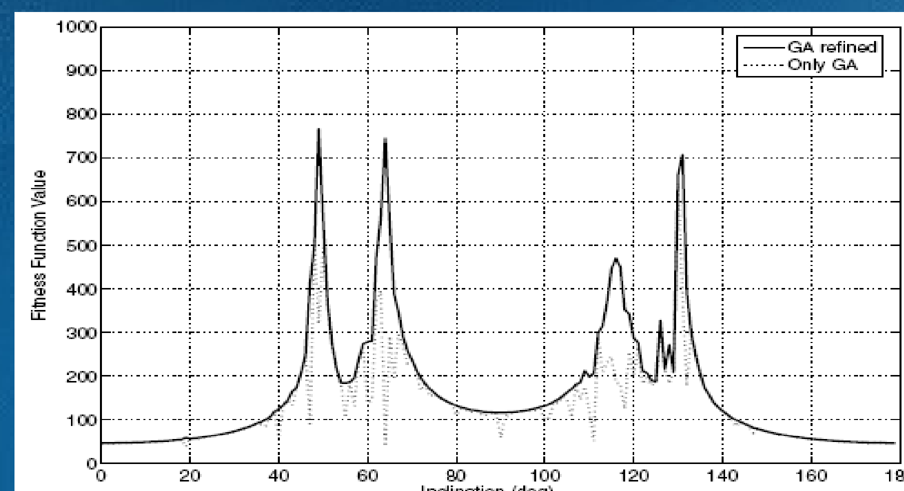
Magic Inclination [5]

The possibility of obtaining a natural periodic relative motion of formation flying Earth satellites is investigated both numerically and analytically. The numerical algorithm is based on a genetic strategy, refined by means of nonlinear programming, that rewards periodic relative trajectories. First, *we test our algorithm using a point mass gravitational model. In this case the period matching between the considered orbits is a necessary and sufficient condition to obtain invariant relative trajectories*.

Then, the J2 perturbed case is considered. For this case, the conditions to obtain an invariant relative motion are known only in approximated closed forms which guarantee a minimal orbit drift, not a motion periodicity. *Using the proposed numerical approach, we improved those results and found two couples of inclinations (63.4 and 116.6 deg, the critical inclinations, and 49 and 131 deg, two new "special" inclinatio… …eriodic relative r…*

$$J(\kappa) = \frac{1}{0.001 + \sqrt{\left(\frac{x_f - x_0}{x_0}\right)^2 + \left(\frac{y_f - y_0}{y_0}\right)^2 + \left(\frac{z_f - z_0}{z_0}\right)^2 + \left(\frac{\dot{x}_f - \dot{x}_0}{\dot{x}_0}\right)^2 + \left(\frac{\dot{y}_f - \dot{y}_0}{\dot{y}_0}\right)^2 + \left(\frac{\dot{z}_f - \dot{z}_0}{\dot{z}_0}\right)^2}}$$





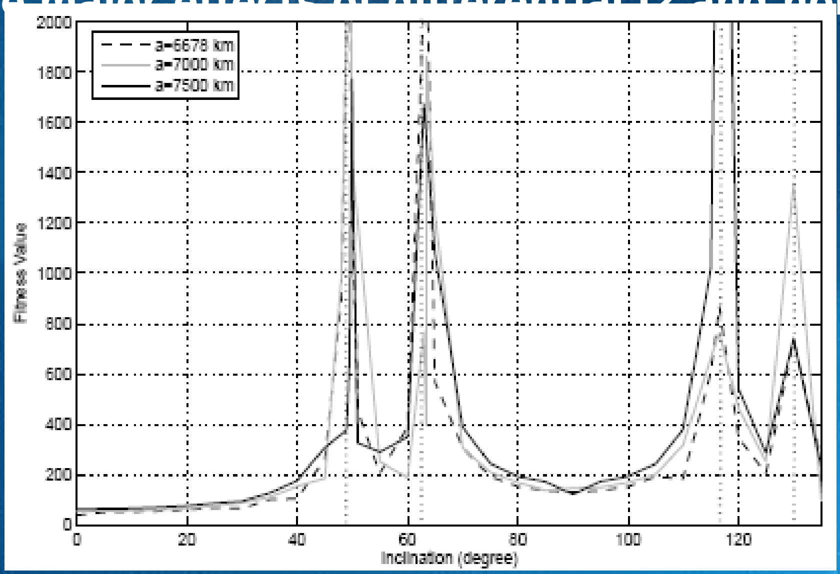
RECENT ADVANCES IN FORMATION FLIGHT - METHODS

Magic inclination [5]

A surprising behaviour is found performing the simulation for given semi-major axis and eccentricity, but varying the inclination. Instead of the expected two peaks of fitness value (corresponding to the critical inclinations), we found other two inclinations, about 50° and its supplementary 180°-50°, which give high values for the fitness.

***This means that for these particular inclinations, the effects of the un-periodicity due to the J2 perturbation are weaker and easier to compensate it's confirmed by the plot in the Figure where it's shown how these inclinations are not sensitive to variations of semi-major axis*.**

This behaviour is probably due to the fact that for the 50° and 130° inclination case, the relative orbits are never really closed, even if the causes of the drifting apart of the formation seem to be weaker. But for large formations these benefits slowly disappear because of the major effects of differential J2 and non linearities. *****



NONLINEAR METHODOLOGIES APPLIED TO THE FORMATION FLIGHT

Analysis of dynamical systems techniques appropriate to the near Earth case and found a family of candidate reference orbits whose nearby orbits support formation flight.

Using Routh reduction and Poincare's section techniques, we can found a fixed stable point for which passes the desired orbit.

ROUTH REDUCTION - THE REDUCED EQUATIONS I

We will use the Routh reduction technique to rewrite the equations of motion. This procedure will enable us to study rst the reduced dynamics in the meridian plane of the satellite before dealing with the dynamics in the longitudinal direction

ρ distance from the origin to a given point (satellite), θ colatitude, Ø longitude. The potential energy including the J2 effect is givne by: (The potential function is the main problem in articial satellite theory).

$$U = -\frac{\mu}{\rho} + \frac{\mu R_e^2 J_2}{\rho^3}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

In order to find the Routhian, the equation is given by

$$R = L - \sum_{i=1}^{n} p_i \dot{q}_i$$

$$R = \frac{1}{2}(\dot{\rho}^2 + \rho^2\dot{\theta}^2) - \frac{H_z^2}{2\rho^2\sin^2\theta} - U(\rho,\theta)$$

$$R = \frac{1}{2}(\dot{r}^2 + \dot{z}^2) - \frac{H_z^2}{2r^2} - U(r,z)$$

The potential energy equation will be:

$$U(r,z) = -\frac{\mu}{(r^2+z^2)^{1/2}} + \frac{\mu R_e^2 J_2}{(r^2+z^2)^{3/2}}\left(\frac{3}{2}\frac{z^2}{r^2+z^2} - \frac{1}{2}\right)$$

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ROUTH REDUCTION - THE REDUCED EQUATIONS II

So, Routhian function becomes:

$$R = \frac{1}{2}(\dot{r}^2 + \dot{z}^2) - \frac{H_z^2}{2r^2} - \left[-\frac{\mu}{(r^2+z^2)^{1/2}} + \frac{\mu R_e^2 J_2}{(r^2+z^2)^{3/2}} \left(\frac{3}{2} \frac{z^2}{r^2+z^2} - \frac{1}{2} \right) \right]$$

The reduced equations are given by:

$$\frac{d}{dt}\left(\frac{\partial R}{\partial \dot{r}}\right) = \frac{\partial R}{\partial r}, \qquad \frac{d}{dt}\left(\frac{\partial R}{\partial \dot{z}}\right) = \frac{\partial R}{\partial z}$$

So, the equations are:

$$\ddot{r} = H_z^2 \frac{1}{r^3} - \mu \frac{r}{(r^2+z^2)^{3/2}} - \frac{3\mu R_e^2 J_2}{2} \frac{r}{(r^2+z^2)^{5/2}} + \frac{15\mu R_e^2 J_2}{2} \frac{r z^2}{(r^2+z^2)^{7/2}}$$

$$\ddot{z} = -\mu \frac{z}{(r^2+z^2)^{3/2}} - \frac{3\mu R_e^2 J_2}{2} \frac{z}{(r^2+z^2)^{5/2}} + \frac{3\mu R_e^2 J_2}{2} \frac{(3z^2 - 2r^2) z}{(r^2+z^2)^{7/2}}$$

$$E = \frac{1}{2}(\dot{r}^2 + \dot{z}^2) + \frac{H_z^2}{2r^2} + U(r,z)$$

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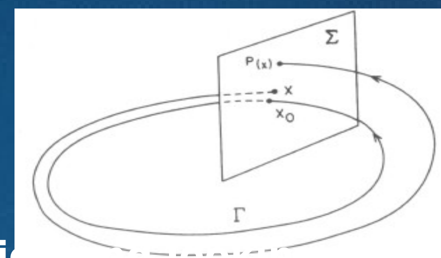
NONLINEAR METHODOLOGIES APPLIED TO THE FORMATION FLIGHT

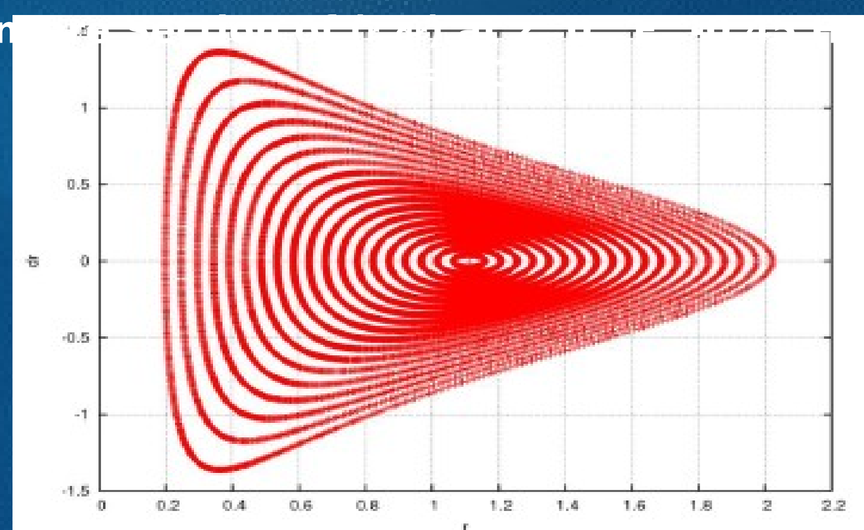
POINCARÉ MAP

After performing Routh reduction, We use the method of Poincare section in finding the initial conditions for orbits that are dynamically favorable to formation flight.



By studying this Poincare secti ... e stable fixed point, We can find the pseudo-circular
orbit (which corresponds to the fixed point in the middle of Figure) whose nearby orbits can be used for formation flight. Clearly, this fixed point corresponds to a periodic orbit in the reduced system.

Poin ... 2=0.3



NONLINEAR METHODOLOGIES APPLIED TO THE FORMATION FLIGHT

Work Done

Analysis of Stable Fixed point depending on energy and z-component of the angular momentum (Hz^2)

Initial conditions

- Triangular Cluster near the Pseudo-Circular Orbit

Methods of monitoring the formation of satellites

- Area
- Relative distances
- Time of formation
- Triangular relationship

Analysis of results depending on z-component of the angular momemtum

Orbital elements

- Relation among Orbital elements, Energy and Hz^2

Useful range for positioning satellite around stable fixed point R=1.1133496883

- Analysis of three formations
- Analysis of three formations in yz-plane

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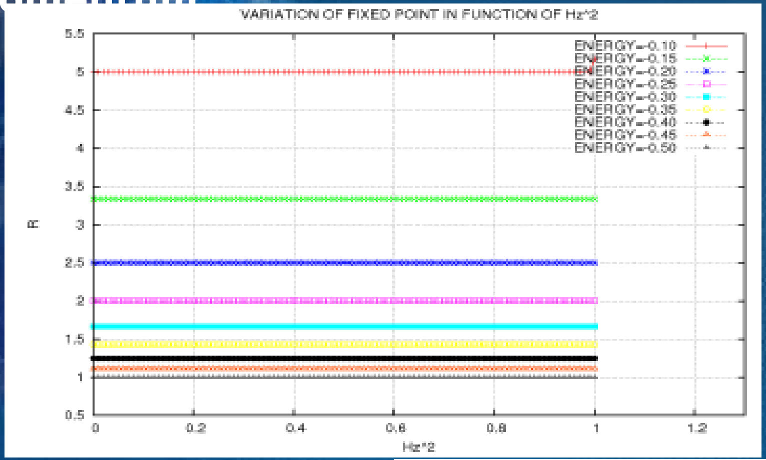
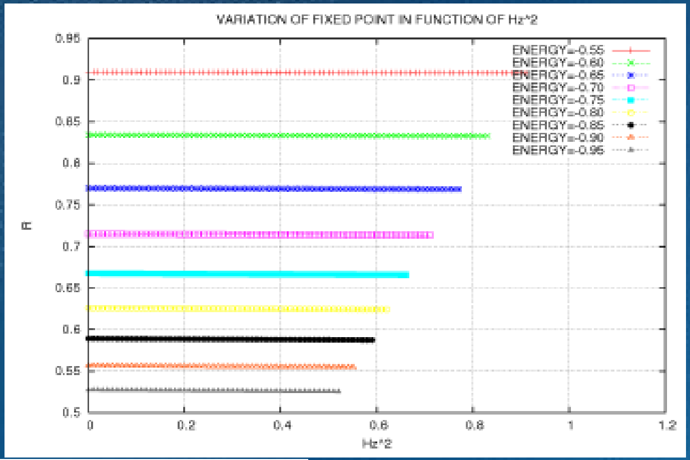
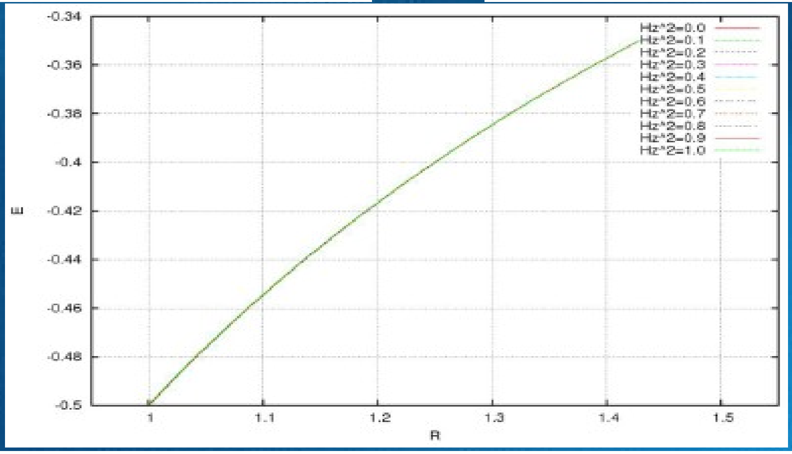
NONLINEAR METHODOLOGIES APPLIED TO THE FORMATION FLIGHT

Analysis of Stable Fixed point depending on energy and z-component of the angular momentum (Hz^2)

Thanks to Routh reduction; We can calculate the stable fixed point knowing values of E and H2z , then We will
make an analysis of the fixed point based on the energy and the H2z **. This analysis is referred to see how the**
distance of fixed point changes in relation to energy and z-component of the angular momentum





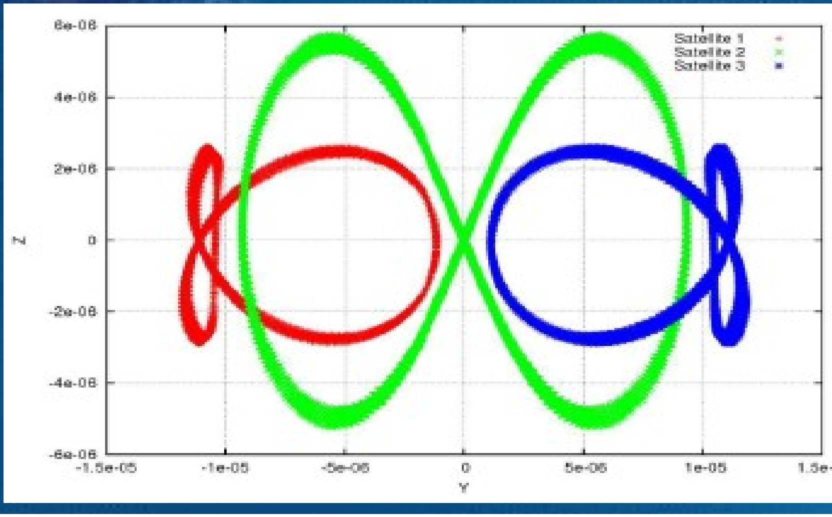


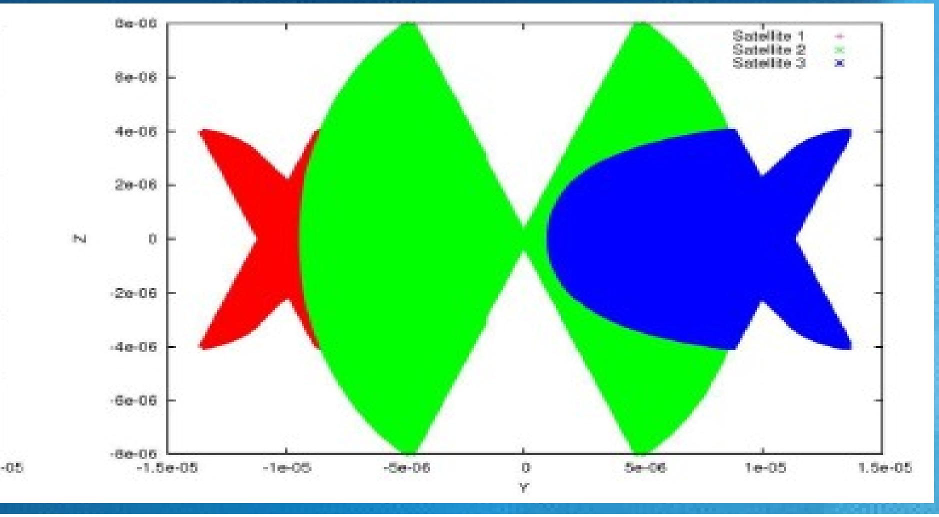
Triangular cluster -Initial conditions

By using the stable fixed point and the points nearby as well as making slight changes in the longitudinal angle (and possibly in the time t), we can construct different kinds of cluster which will remain together after many years (corresponding to thousands of revolutions around the Earth**). i.e. if we fix E = -0.45, Hz^2= 0.3 (for example), the fixed point for the Poincare section at z = 0 will be (rf ; 0) where r=1:11133496883 (about 710 km above the Earth). The following initial conditions will give a triangular cluster (with each side close to 100 meters, approximately).**

r	$\dot{r}$	ϕ	t
1.11133496883	0.0	$1x10^{-5}$	$1x10^{-5}$
1.11134196883	0.0	0.0	$1x10^{-5}$
1.11133496883	0.0	$-1x10^{-5}$	$1x10^{-5}$

The evolution of these three satellites in a triangular cluster were plotted in a frame whose origin is at their instantaneous barycenter, with the yz-plane orthogonal to the line of sight, the x-axis pointing towards the center of the Earth, and the y-axis and the z-axis pointing towards the (instantaneous) west and north respectively.

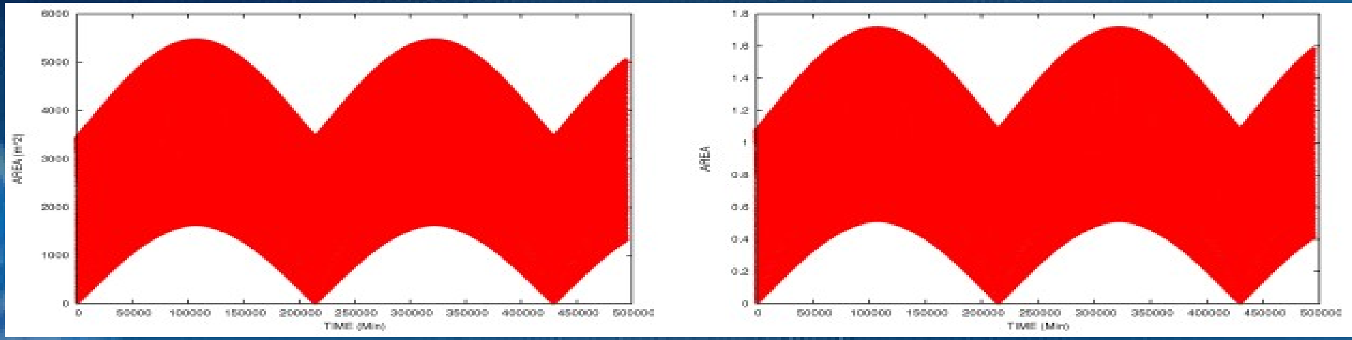




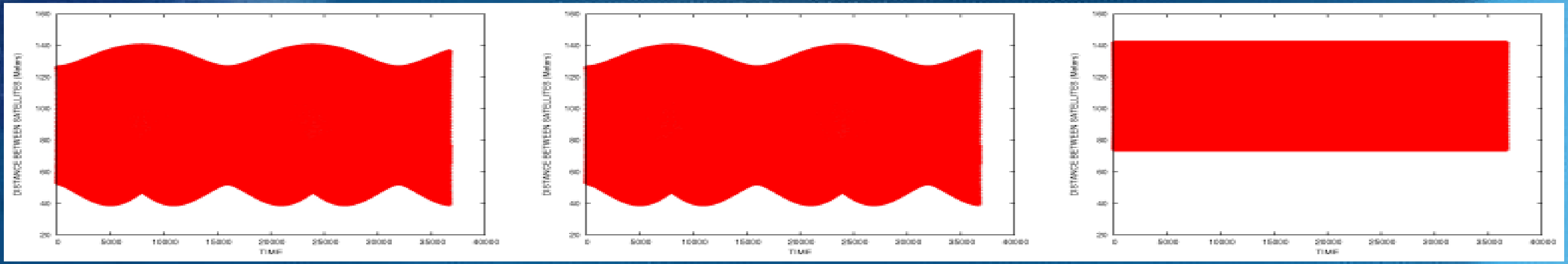
NONLINEAR METHODOLOGIES APPLIED TO THE FORMATION FLIGHT

Analysis of the formation of satellites

Area: The triangle's area formed by the cluster of satellites will be calculated for knowing how the area changes along time.



Relative distances:

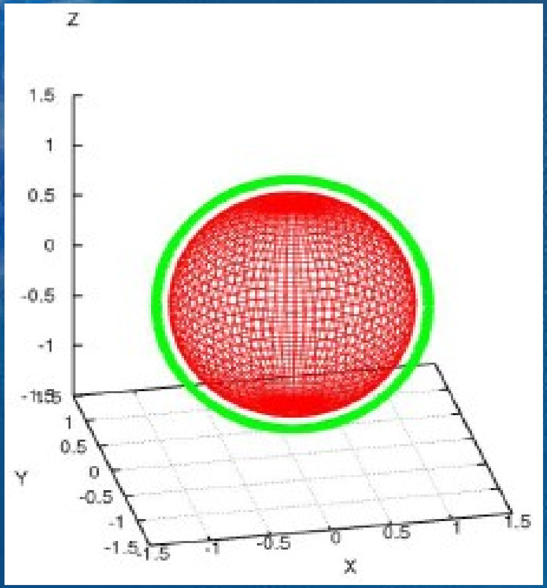


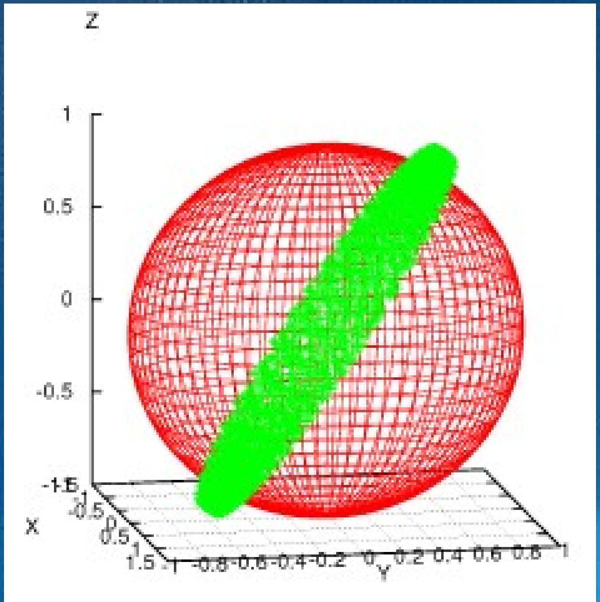
Time of formation: It is going to calculate the time that the satellites remain within a hypothetical sphere of certain radius. The hypothetical sphere has its center in the barycentre of the triangle formed by the satellites

r_{sphere} (m)	Time (TU)	Satellite	N^o Rev.	Distance (m)
70	0.0	1-3	0	72.67624047
75	0.4	1	0	75.3748043
80	1478.4	3	198.7815	80.0333474
85	3384.0	3	455	85.0107022
90	6926.8	1	931.358	90.0046592

Orbital elements

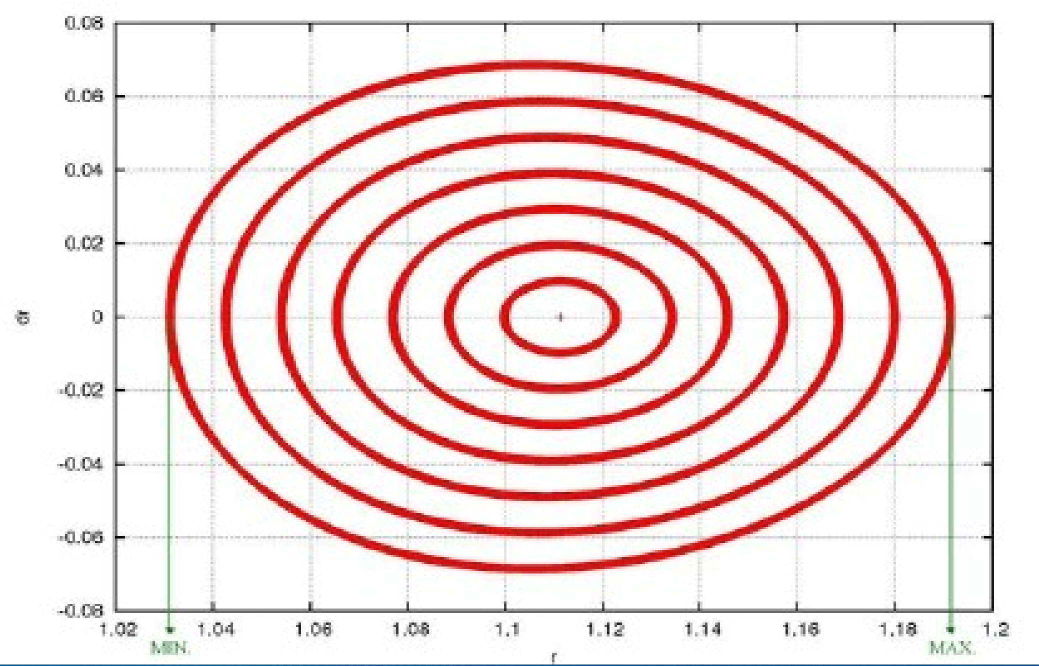
r	Altitude (Km)	Eccentricity	Inclination (Degrees)
1.11133496883	703.632-717.350	0.00031-0.000929	58.678-58.712





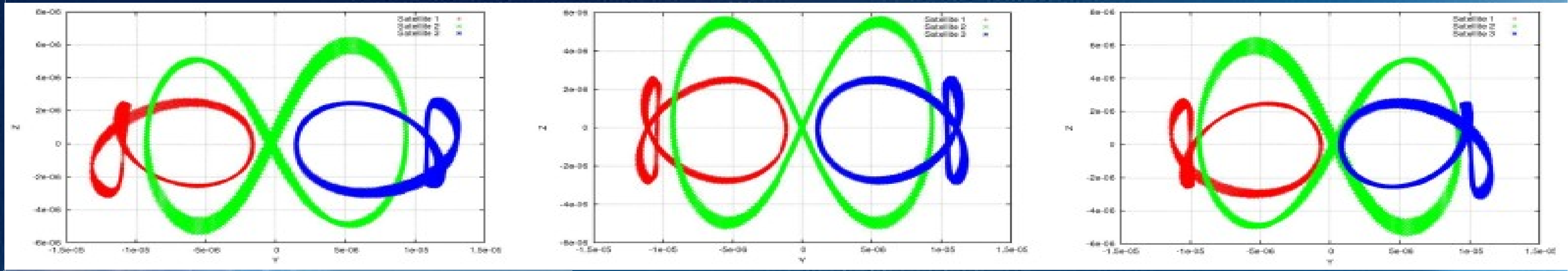
NONLINEAR METHODOLOGIES APPLIED TO THE FORMATION FLIGHT

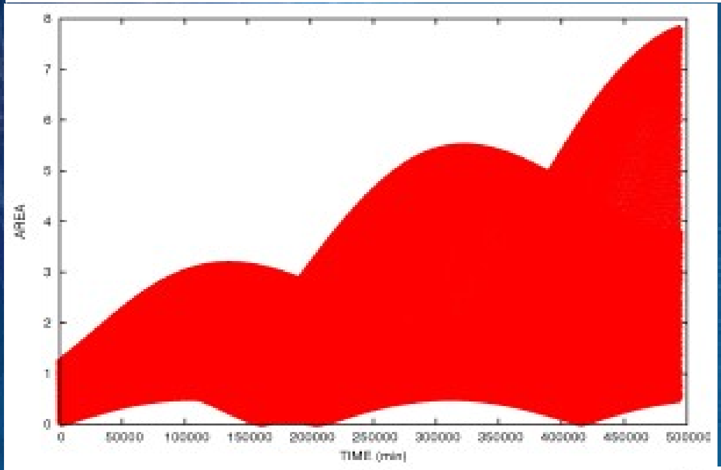
Useful range for positioning satellite around stable fixed point R=1.11133496883

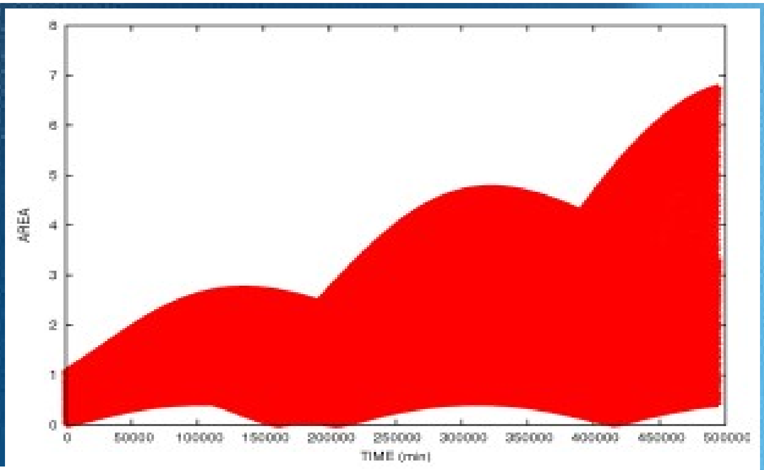


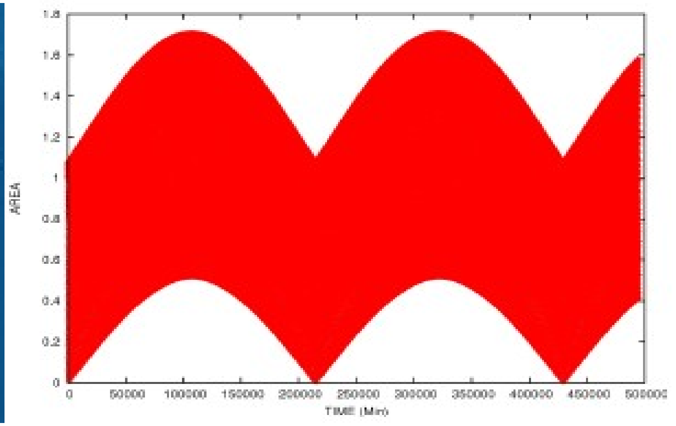
Positions	r(altitude)	Minimum altitude flight	Time (TU)
Minimun	1.03125 (200Km)	1.030949 (198.07Km)	7690.10622
Fixed point	1.1113349688 (712.54km)	1.11098 (710.27km)	75.41
Maximun	1.1914199377 (1225.09Km)	1.0309485 (198.07km)	7701.14652

NONLINEAR METHODOLOGIES APPLIED TO THE FORMATION FLIGHT









REFERENCES

[1] Sedwick, R.J., E.M.C. Kong and D.W. Miller, "Exploring orbital dynamics and micropropulsion for aperture synthesis using distributed satellites systems: applications to techsat21". AIAA 52-89; 1998.

[2] Schaub, H. and K.T. Alfriend. "J2 invariant reference orbits for spacecraft formations". Flight Mechanics Symposium, Goddard Space Flight Center, May 18-20; 1999.

[3] Brouwer, D. "Solution of the problem of articial satellite theory without drag". The Astronautical Journal 64, No. 1274, 478-397; 1959.

[4]Dong-Woo Gim and Kyle T. Alfriend "Satellite relative motion using dierential equinoctial elements". Celest. Mechanics and Dynamical Astronomy 92:295-336; 2005.

[5] Sabatini M, D. Izzo and R. Bevilacqua "Special Inclinations Allowing Minimal Drift Orbits for Formation Flying Satellites". Journal of guidance, control and dynamics 31, No. 1; 2008.

[6] Tan, Z., Bainum, P.M., “An improved Strategy for Maintaining Distance Between Satellites in an Elliptically Orbiting Constellation” Paper AAS 01-219, *AAS/AIAA Space Flight Mechanics Meeting*, Santa Barbara (California), 11-14, February 2001.

NOTE: (Figures)

TFC: Estratègies per mantenir separació constant entre satèl·lits situats en constel·lacions d’òrbites el·líptiques.

Autor: Meritxell Viñas Tió; Data: 27 de febrer de 2006

[7]Koon, W.S., J.E. Marsden, J.J. Masdemont and R.M. Murray. "J2 Dynamics and formation ight". AIAA-2001-4090; 2001