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SOLAR SAIL SURFING ALONG FAMILIES OF EQUILIBRIUM POINTS

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Abstract

We have considered the planar circular RTBP + solar radiation pressure to model the movement of a solar sail in the Earth - Sun system. It is known that for a fixed value of the sail lightness number this model has three 1-parametric families of equilibria parametrised by the sail orientation. Most of these fixed points are unstable and require a control strategy to keep a sail close to them. We have studied the linear dynamic around them and how it varies when the sail orientation is changed. We have used this information to derive strategies to move along these families and control the trajectory of the sail close to a given fixed point. Finally, we have tested our strategies for a particular mission.

1 Introduction

Solar Sailing is a proposed form of spacecraft propulsion using large membrane mirrors. The impact of the photons emitted by the Sun on the surface of the sail and their further reflection produce momentum on it. Although the acceleration produced by this reflection is smaller than the one achieved by a ‘traditional’ spacecraft it is continuous and unlimited. These makes long term missions more accessible. As it can be seen in the literature[4, 3, 6, 8, 1] solar sails open a wide range of new mission.

It is well know that a solar sail is an orientable surface, the orientation is defined by the pitch angle (α). Another important parameter is the sail lightness number (β) used to define the sail's effectiveness. In this paper we have considered a flat and perfectly reflecting sail, so the force due to the solar radiation pressure is normal to its surface.

To model the dynamics of the sail we have taken the Sun - Earth Planar Restricted Three Body Problem (RTBP) and added the solar radiation pressure effect. This model is a perturbation of the RTBP and depends on two parameters, the sail lightness number β and the sail orientation α. It is well know that the RTBP has five equilibrium points ($L_{1,\dots,5}$). For a small β these five equilibrium points are replaced by five continuous families of equilibria parametrised by the sail orientation. As β increases 4 of these families connect and then the fixed points are displaced in three 1-parametric families[4, 5].

We can classify the equilibrium points by their stability. In this paper we will focus on the unstable fixed points that have one pair of real eigenvalues and a pair of complex eigenvalues. We will discuss strategies[2] to control the trajectory close to one of these unstable points and we will apply them to move the trajectory around these families of equilibria.

Let p_0 be a fixed point for $\alpha = \alpha_0$. Take $\vec{v}_1, \vec{v}_2$ the stable and unstable eigenvectors and $\vec{v}_3, \vec{v}_4$ the real and complex part of the complex eigenvectors. It is well know that $V = \{\vec{v}_i\}_{i=1,\dots 4}$ are a basis and the couple $\{p_0; V\}$ is a reference system. This reference system will help us understand the linear dynamics around the equilibrium point and will be very used on our strategies.

When the probe is close to the fixed point p_0, the trajectory escapes along the unstable direction

($\vec{v}_1$). Changes on the sail orientation will produce changes on the fixed point and its eigendirections. If we want to control the trajectory close to p_0 we need to change the sail orientation so that the new unstable direction brings the trajectory close to p_0. If we want to move the trajectory along the family of equilibrium points we need to change the sail orientation so that the sequence of unstable directions takes the probe along this family.

We have designed a toy mission to show how these techniques work. We have supposed that our sail is close to an unstable equilibrium point displaced 5^o from the Sun - Earth line and we want to move the probe along the family of equilibria until we reach a fixed point at 10^o from the Sun - Earth line. When we reach the final destination a control strategy has been used to control the sail close to the final point for 10 years.

2 Equations of Motion

To describe the dynamics of the solar sail we have supposed that Earth and Sun are point masses moving around their mutual centre of mass. The sail is under the gravitational effect of both bodies and the solar radiation pressure. The units of mass, distance and period have been normalised so that the total mass of the system is 1, the Sun - Earth distance is 1 and the period of rotation is 2π. With these units the gravitational constant is also 1. We use a rotation reference system so that Earth and Sun are fixed on the x axis.

The radiation pressure depends on the position of the sail, its orientation and the characteristic acceleration. The sail orientation is given by the pitch angle α: the angle between the Sun - line and the normal vector to the surface of the sail ($\vec{n}$). As the sail cannot point towards the Sun we have that $\alpha \in [-\pi/2, \pi/2]$. Let us define ϕ as the angle that gives the position of the sail with respect to the Sun on the $\{x, y\}$ - plane taking the origin of angles on the right hand side of the Sun (see Figure 1). Then $\vec{n} = (\cos(\phi + \alpha), \sin(\phi + \alpha))$.

In this paper we will suppose that the sail is flat and perfectly reflecting, then:

$$F_{sail} = \beta \frac{1-\mu}{r_{PS}^2} \cos^2 \alpha \cdot \vec{n},$$

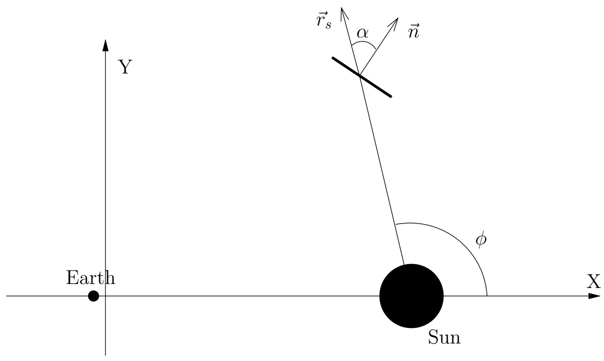

Figure 1: **Schematic representation of the sail orientation α and the angle ϕ.**

where β is the sail lightness number.

In the rotating frame the equations of motion are,

$$\begin{aligned} \ddot{x} &= 2\dot{y} + x - (1-\mu)\frac{x-\mu}{r_{PS}^3} \\ &\quad - \mu \frac{x+1-\mu}{r_{PT}^3} + \kappa \cos(\phi + \alpha), \\ \ddot{y} &= -2\dot{x} + y - \left(\frac{1-\mu}{r_{PS}^3} + \frac{\mu}{r_{PT}^3} \right) y \\ &\quad + \kappa \sin(\phi + \alpha), \end{aligned} \tag{1}$$

where $\kappa = \beta \frac{1-\mu}{r_{PS}^2} \cos^2 \alpha$.

Notice that the equations depend on two parameters: β and α. If $\beta = 0$ or $\alpha = \pm\pi/2$ the equations are the same as in the RTBP.

3 Equilibrium Points

It is well known that the RTBP[7] has five equilibrium points $L_{1,\dots,5}$. For small values of β these five points are replaced by five families of equilibrium parametrised by the sail orientation α. As β increases 4 of these families will merge having the fixed points displaced in 3 curves parametrised by α.

As it can be seen in Figure 2, for the fixed values of β considered here, there is one curve C_1 containing L_2 at the left hand-side of the Earth, another curve C_2 containing $L_{1,3,4,5}$ that surrounds the Sun and leaves the Earth on its left and a third curve C_3 similar to C_2 but closer to the Sun. As β increases C_3 comes closer to the Sun.

McInnes[4] has shown that the equilibrium points of

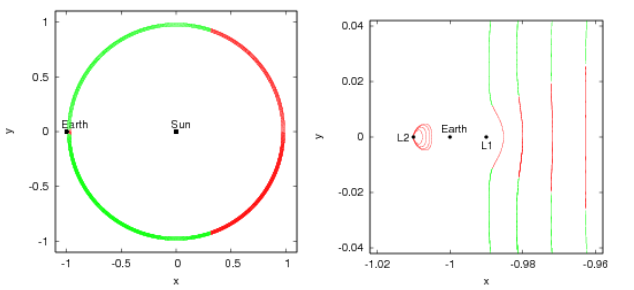

Figure 2: **Equilibrium points for different values of β. In red the T_1 points and in green the T_2 points.**

the system are in general unstable. We can see that in the plane the equilibrium points can be classified in two classes depending on the eigenvalues of the differential matrix at a fixed point. The first class (T_1) includes the equilibrium points with one pair of real eigenvalues and one complex eigenvalue and the second class (T_2) has two pairs of complex eigenvalues. In Figure 2 we can see the relation between these point on the plane.

In this paper we will focus on the dynamic close to equilibrium points of the first class. We want to understand the natural dynamics around these points and how it varies when the sail orientation is changed in order to control the dynamics at our will.

3.1 Linear dynamic around the equilibrium points

As it has already been mentioned the equilibrium points of the first class have one pair of real eigenvalues ($\pm\lambda$) and one pair of complex eigenvalues ($\nu \pm \mathtt{i}\omega$). Although ν can be different from zero, on the cases that we are considering it will be small, as a first approximation we will consider $\nu = 0$. This means that the linear dynamic around the equilibrium point is saddle $\times$ centre. The real effect will be considered later during the simulations.

Let p_0 be a fixed point for α_0 and take the reference system $R_0 = \{p_0; \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$. Where $\vec{v}_1, \vec{v}_2$ are the unstable and stable eigenvectors respectively and $\vec{v}_3, \vec{v}_4$ are the real and imaginary part of the complex eigenvectors. When the probe is close to p_0 the linear dynamics is the one that matters and the trajectory can be described in terms of R_0. The trajectory of the probe will escape along the unstable direction and rotates on its central projection as can be seen in Figure 3.

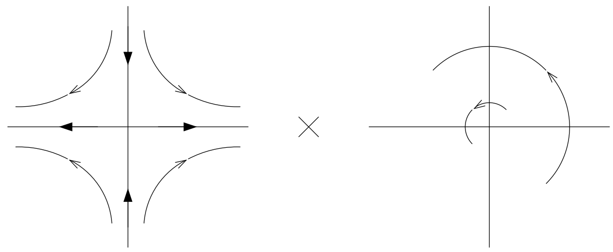

Figure 3: **Schematic representation of different trajectories of the probe on the reference system $R_0 = \{p_0; \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$.**

When the sail orientation is changed the fixed point varies and so do the eigenvectors. We want to understand the effect of these changes to the trajectory of the probe, i.e. we have a different reference system.

Let p_1 be a new fixed point for α_1 and suppose that $|\alpha_0 - \alpha_1|$ is small. The new eigenvectors will be close to $\vec{v}_i$ $i = 1, \ldots, 4$. When the sail orientation is changed from α_0 to α_1 the trajectory of the probe will change from escaping along the unstable direction of p_0 to escaping along the unstable direction of p_1. If we choose an appropriate sail orientation we can: make the trajectory stay close to one of these equilibrium points or make the trajectory move along this family.

3.2 Linear approximation of the surface of equilibria

As in the RTBP we do not have an explicit expression for the equilibrium points $p(\alpha)$. As changes in the sail orientation will be small the linear approximation of these curves around a fixed point will be enough.

If $p(\alpha_0)$ is the position of the equilibrium point for a fixed angle α_0, it is a known fact that the linear approximation is given by,

$$p(\alpha) = p(\alpha_0) + \left.\frac{\partial p}{\partial \alpha}\right|_{\alpha_0} (\alpha - \alpha_0). \qquad (2)$$

It is easy to see that $\frac{\partial p}{\partial \alpha}$ can be computed by solving,

$$D_x f(p_0, \alpha_0) \frac{\partial p}{\partial \alpha} = -\frac{\partial f}{\partial \alpha}(p_0, \alpha_0),$$

where $f(x, \alpha)$ is the function defining the flow.

4 Surfing through the family of equilibria

As it has been mentioned before we want to use the invariant manifolds to control the trajectory of the sail close to an unstable equilibrium point and move around the family of unstable equilibrium points. We will start discussing how to control the trajectory of a probe around an equilibrium points and we will extend these ideas to move around the family of equilibria.

4.1 Control around a fixed point

In this section we will focus in controlling the trajectory of a probe around an unstable equilibrium point. This strategy has already been tested in some particular missions[2].

Let p_0 be a fixed point for a given sail orientation $\alpha = \alpha_0$. As it has already been said in section 3.1 when the probe is close to p_0 its trajectory will escape along the unstable direction $\vec{v}_1$. We want to find a new sail orientation $\alpha = \alpha_1$ such that the unstable direction of the new fixed point p_1 brings the trajectory close to p_0 (see Figure 4). When the trajectory is close to p_0 we will restore the sail orientation to $\alpha = \alpha_0$ and repeat this process. It is important to note that the projection on the central plane of the trajectory can grow: as the central behaviour are rotations around the different equilibrium points and this process can result unbounded. For this reason we have to be careful when we choose the new sail orientation.

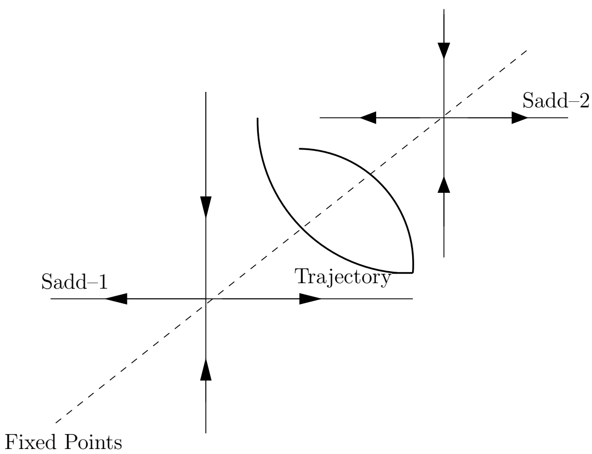

Figure 4: **Schematic representation of the idea of controlling the instability due to the saddle.**

From now on the sail trajectory will be described in term of the reference system $R_0 = \{p_0; \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ (see section 3.1). Hence, in this reference system the trajectory of the probe is:

$$z(t) = \sum_{i=1}^{4} s_i(t) \cdot \vec{v}_i,$$

where $s_{1,2}(t)$ describes the projection of the trajectory on the saddle plane and $s_{3,4}(t)$ its projection on the centre plane. As time goes by $s_1(t) \to \infty$, $s_2(t) \to 0$ and $\sqrt{s_3^2(t) + s_4^2(t)}$ is constant.

For our control strategy we need to define the parameters ε_{max} and ε_{min}, were $0 < \varepsilon_{min} < \varepsilon_{max} << 1$ define the region of motion around the equilibrium point on the saddle projection. To find the position of the new fixed point we will use the parameter d, see Figure 5.

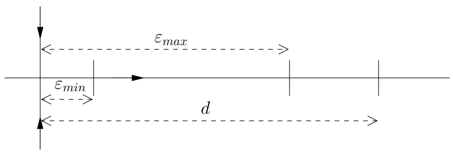

Figure 5: **Schematic representation of the parameters $\varepsilon_{max}, \varepsilon_{min}$ and d on the saddle plane.**

When $|s_1(\tau_1)| \geq \varepsilon_{max}$ we will change the sail orientation, i.e. the fixed points position. We need $u_1 > s_1(t_*)$ for the trajectory to come back close to p_0. We will take $u_1 = d$ with $d \in (\varepsilon_{max}, M)$ (M is an upper bound for the linear approximation of the dynamics around p_1). The new sail orientation ($\alpha = \alpha_1$) is found by solving equation (2). In the reference system R_0 equation (2) is:

$$\hat{p}(\alpha) = \left.\frac{\partial \hat{p}}{\partial \alpha}\right|_{\alpha=\alpha_0} (\alpha - \alpha_0). \tag{3}$$

Hence,

$$\alpha_1 = \alpha_0 + \frac{u_1}{(\partial \hat{p}/\partial \alpha)_1},$$

and

$$u_i = (\partial \hat{p}/\partial \alpha)_i \left(\frac{u_1}{(\partial \hat{p}/\partial \alpha)_1} \right), , \qquad i = 2, 3, 4. \tag{4}$$

Then the sail orientation is changed and $s_1(t) \to 0$, when $|s_1(\tau_2)| \leq \varepsilon_{min}$ the sail orientation will be restored $\alpha = \alpha_0$ and this process is repeated. Notice that this is true if the stable and unstable direction of p_0 and p_1 do not vary much w.r.t each other. As the variation of the sail orientation will be small the eigenvectors will vary slightly.

It can be seen that if this process is repeated over and over $s_1(\tau_{2i+1})$ stabilises, so the sail orientation tends to change only between two fixed values.

The saddle projection of the motion can be described by the following equation:

$$\left.\begin{array}{rcl} s_1(t) & = & \hat{u}_1(\alpha) + (s_{10} - \hat{u}_1(\alpha))e^{\lambda(\alpha)t} \\ s_2(t) & = & \hat{u}_2(\alpha) + (s_{20} - \hat{u}_2(\alpha))e^{-\lambda(\alpha)t} \end{array}\right\}, \quad (5)$$

where $(\hat{u}_1(\alpha), \hat{u}_2(\alpha))$ are the different equilibrium point depending on α and (s_{10}, s_{20}) is the position of the probe when the α is changed. It can be checked that the time it takes to go from ε_{min} to ε_{max} is:

$$\Delta t_1 = \frac{1}{\lambda(\alpha)} \log\left(\frac{\varepsilon_{max}}{\varepsilon_{min}}\right),$$

and the time to go from ε_{max} to ε_{min} is:

$$\Delta t_2 = \frac{1}{\lambda(\alpha)} \log\left(\frac{u_1 - \varepsilon_{max}}{u_1 - \varepsilon_{min}}\right).$$

These are some good estimation of the time between manoeuvres. As we can see these times mainly depend on ε_{max}, ε_{min} and d, which can be changed to satisfy certain constraints if needed.

Let us now focus on the dynamic on the central behaviour. As it has already been said the central projection of the trajectory will be a sequence of rotations around the different equilibrium points. The angle of rotation will depend on $\Delta t_1, \Delta t_2$ and ω, the imaginary part of the complex eigenvalue.

Lemma: *The composition of rotations of angle θ_1 and θ_2 around two fixed points x_1 and x_2 is a rotation of angle $\theta_1 + \theta_2$ around a new point x_3, whose position depends on θ_1, θ_2, x_1 and $x2$.*

This lemma shows that if we rotate around two different equilibrium points the trajectory will be bounded as the points where the sail orientation is changed will be a rotation around a different fixed point. In Figure 6 we can see where these points will be placed. It is easy to see that their position will depend on the position of the probe at the beginning of the trajectory and on the rotation angles θ_1 and θ_2.

The two angles θ_0 and θ_1 depend on $\varepsilon_{max}, \varepsilon_{min}$ and d. We will change ε_{max} and ε_{min} in an appropriate

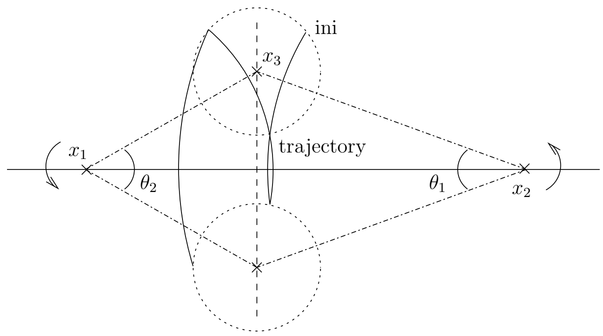

Figure 6: **Schematic representation of the trajectory on the centre projection.**

way to reduce the projection of the trajectory on the centre manifold. This will only be done if this projection is big and the linear approximation is not good enough.

4.2 Moving along the family of equilibrium points

In this section we will explain how to move in a controlled way along the family of unstable equilibrium points using similar ideas to the previous section.

Now we want to go from the vicinity of a fixed point p_0 to another point p_f in a controlled way. We want to be able to control the probe once we reach p_f.

We recall that when the probe is close to the equilibrium point p_0 the trajectory escapes along the unstable direction. We want to find a sequence of changes of the sail orientation α_i so that the trajectory keeps close to the family of equilibria. The idea is that the unstable directions of the equilibrium points p_i takes the trajectory from the vicinity of one point to the other (see Figure 7). The main idea is to use a variation of the control strategy described before but moving the reference system.

Let p_i be a sequence of fixed points for the sequence of sail orientation α_i. For each p_i we will consider the reference system $R_i = \{p_i; \vec{v}_{i1}, \vec{v}_{i2}, \vec{v}_{i3}, \vec{v}_{i4}\}$ where as before $\vec{v}_{i1}, \vec{v}_{i2}$ are the unstable and stable eigenvectors of p_i and $\vec{v}_{i3}, \vec{v}_{i4}$ are the real and imaginary part of the complex eigenvectors.

Let us suppose we are close to p_0 and we want to go to p_f. First of all we need to know the relative position of p_f w.r.t p_0 in the saddle plane. We need this information to know the direction of the sequence

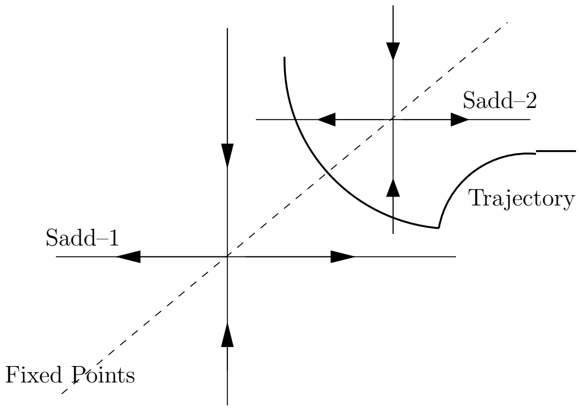

Figure 7: **Schematic representation of the saddle projection of the trajectory.**

of fixed points. Then we must use a modified control strategy to the one explained in section 4.1 to place the trajectory in the desired side of the saddle. Usually we will make some control to start with a small enough central projection.

As before we need to define the parameters ε_{max}, ε_{min} and d, where again $0 < \varepsilon_{min} < \varepsilon_{max} << 1$ define the region of motion around the equilibrium point on the saddle projection. Now to find the position of the new fixed point we need $d \in (\varepsilon_{min}, \varepsilon_{max})$. In Figure 8 we can see a schematic representation of these values.

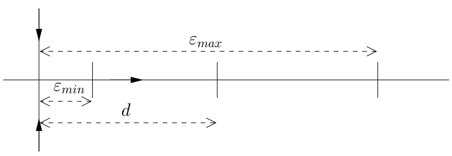

Figure 8: **Schematic representation of the parameters $\varepsilon_{max}, \varepsilon_{min}$ and d on the saddle plane.**

Once the trajectory is been placed on the appropriate side on the saddle plane we can start the transfer from p_0 to p_f. As before the trajectory will be seen on terms of the reference system R_0 so

$$z(t) = \sum_{i=1}^{4} s_i(t)\vec{v}_i.$$

When $|s_1(\tau_1)| \geq \varepsilon_{max}$ we will change the sail orientation. Now we need the new fixed point to satisfy $u_1 = d$ with $d \in (\varepsilon_{min}, \varepsilon_{max})$. As before the new sail orientation is found by solving equation (2) and

$$\alpha_1 = \alpha_0 + \frac{u_1}{(\partial \hat{p}/\partial \alpha)_1}.$$

Now we must change the reference system from R_0 to R_1 and repeat the process until the trajectory comes close to p_f.

If we focus on the central behaviour we will have rotations around the different equilibrium points. This rotations can also result unbounded. Notice that if the changes between the fixed points are small we will be slightly moving the fixed point and we will end up having a spiralling trajectory on the central projection (see Figure 9). Nevertheless, if the central behaviour starts to grow we can perform a control strategy around the equilibrium point to reduce the central projection.

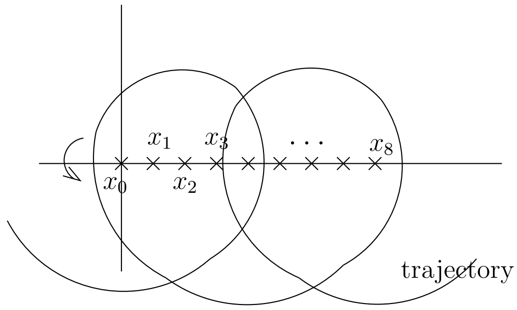

Figure 9: **Schematic representation of the centre projection of the trajectory.**

As before equation (5) describes the movement around the different equilibrium points on the saddle projection and can be used to estimate the time between manoeuvres. Now

$$\Delta t = \frac{1}{\lambda(\alpha)} \log\left(\frac{\varepsilon_{max}}{u_1 - \varepsilon_{max}}\right).$$

Notice that the time between manoeuvres now just depends on the parameter d, the position of the new fixed point and ε_{max}. These parameters can be changed to control the speed in which we move around these family. But we must be careful not to take very big values for ε_{max} as we can we can blow the central part or the linear approximation.

5 Results for a concrete application

To test our algorithms we have considered a sail with a characteristic acceleration $a_0 = 0.3\text{mm/s}^2$. This is the same a_0 that has been used to study the Geostorm Warning Mission[4,8,3]. We will start with the probe close to an unstable equilibrium point p_0 displaced 5^o from the Sun - Earth line at 3003461.3km from the Earth. We want to transfer the trajectory from the neighbourhood of p_0 to the vicinity of a fixed point p_1 displaced 10^o from the Sun - Earth line at 3027723.26km from the Earth. Once the sail is close to p_1 we will use the control strategy described in section 4.1 to maintain the sail close to p_1 for 10 years. In Figure 10 we have a schematic representation of this mission.

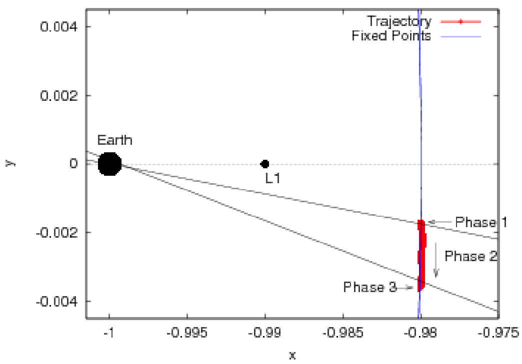

Figure 10: **Schematic representation of the proposed mission.**

Our mission is divided in three phases. In phase 1 we control the sail's trajectory in a vicinity of p_0 and prepare it for phase 2. In the second phase the sail orientation is changed so that the invariant manifolds take the probes trajectory to a vicinity of p_1 in a controlled way. Finally, in phase 3 we will use the control strategy to maintain its trajectory close to p_1 up to 10 years.

We have taken random initial conditions close to p_0. In Figure 11 we can see the final trajectory followed by the sail. The trajectory is plotted in different colours depending on the phase of the mission.

The first phase is need to place the sail in the appropriate side of the saddle. In this case the initial condition is placed on the opposite side and we need to apply some control. In Figure 12 we can see the variation in the sail orientation during the first phase. Figure 13 shows the saddle and centre projection of the trajectory followed by the probe on this phase. We can clearly see how the probe is moved to the correct side of the saddle and bounces from one side to the other until the central movement is small enough to start phase 2.

It takes 2.36 years to go from p_0 to p_1. We need to

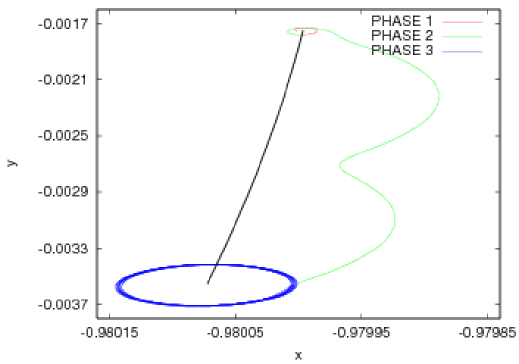

Figure 11: **The trajectory followed by the sail. In red the first phase of the trajectory, in green the second phase and in blue the third phase.**

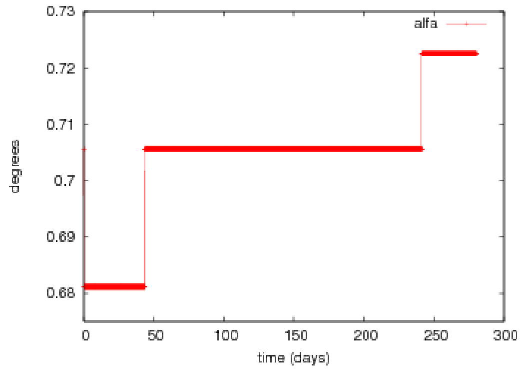

Figure 12: **Phase 1: Variation of the sail orientation w.r.t time.**

do changes in the sail orientation every 17.8 days. These changes are always around 0.02^o. In Figure 14 we can see the variation on the sail orientation w.r.t time.

In Figure 15 we can see the projection of part of the trajectory on the saddle and centre planes defined by R_0. Notice that in the saddle projection we can see the saddle type motion for the different equilibrium points. In the centre projection we can see how the trajectory rotates around the different equilibrium points. In this plot we have used two different colours to illustrate the different sail orientation.

For this particular case we have considered $\varepsilon_{min} = 5 \cdot 10^{-5}$, $\varepsilon_{max} = 10^{-4}$ and $d = 2.5 \cdot 10^{-5}$. If we want the transfer trajectory of the probe to come closer to the equilibrium points we must choose a smaller ε_{max} but this will make the transfer time increase.

When the probe reaches the desired 10^o phase 3 starts. During the control strategy the time be-

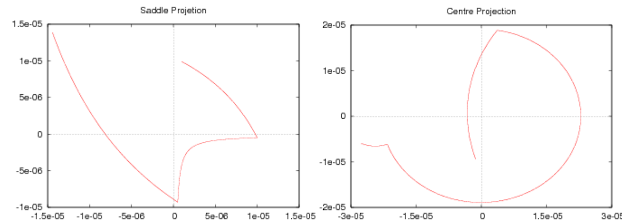

Figure 13: **Phase 1: From left to right projections of the sail's trajectory on the saddle plane ($\vec{v}_1, \vec{v}_2$) and on the centre plane ($\vec{v}_3, \vec{v}_4$).**

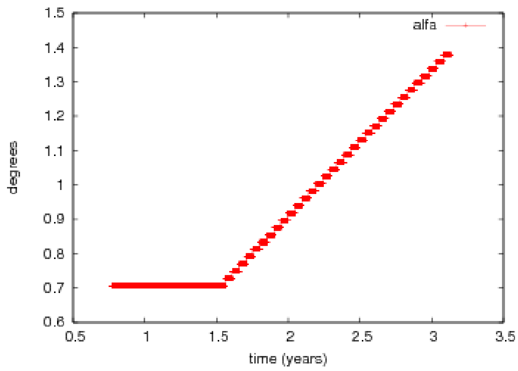

Figure 14: **Phase 2: Variation of the sail orientation w.r.t time.**

tween manoeuvres oscillates between 136 and 40 days. This difference is due to the choice of the different fixed points as one the probe escapes from one fixed point quicker than from the other. The angle variation in the sail orientation varies around the 0.016^o. In Figure 16 we can see the variation of the sail orientation during the control.

Figure 17 show the projection of the trajectory on the saddle and centre planes respectively. We have plotted the trajectory in two colours depending on the sail orientation. In red when p_1 is the fixed point and green when the appropriate sail orientation is chosen. We can see clearly the bouncing between the saddles in the saddle projection. On the centre projection we can see how the trajectory is maintained bounded. In this case no extra manoeuvres to reduce the central behaviour have been done.

6 Conclusions

In this paper we have shown a control strategy to maintain a solar sail close to an unstable equilibrium point. We have also developed a strategy

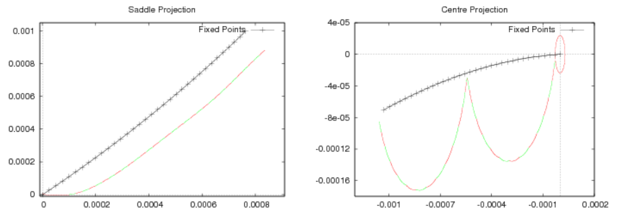

Figure 15: **Phase 2: From left to right projections of the sail's trajectory on the saddle plane and on the centre plane.**

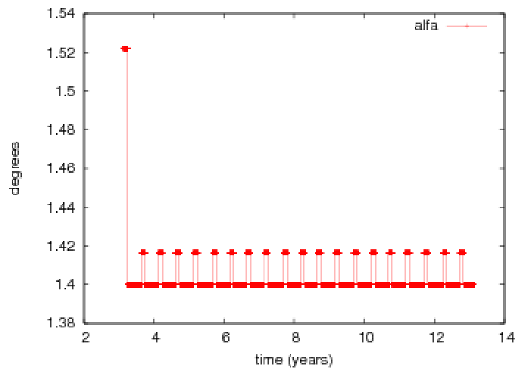

Figure 16: **Phase 3: Variation of the sail orientation w.r.t time.**

to move around the family of unstable equilibrium points.

These strategies have been applied to go from one region to another. As a particular case we have chosen to go from a fixed point displaced 5^o on the Sun - Earth line to a fixed point at 10^o of this line, but these techniques are valid to move around the family of unstable equilibrium points.

They are based on using the natural dynamics of the system. Understanding how the invariant manifolds vary as the sail orientation is changed has been the key. In this work we have only used the information given by linear dynamics. This is useful if we want to be close to the equilibrium points, but might be a constraint in some cases. We could use higher order terms to go further away of the equilibrium point.

7 Acknoledgments

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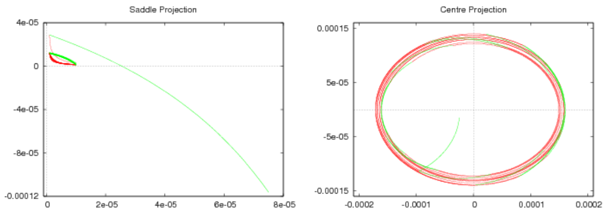

Figure 17: **Phase 3: From left to right projections of the sail's trajectory on the saddle plane and on the centre plane.**

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