ZERO RELATIVE RADIAL ACCELERATION CONES AND CONTROLLED MOTIONS SUITABLE FOR FORMATION FLYING[0]

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Assume that a constellation of satellites is required to flight close to a given nominal trajectory and that there is some freedom in the selection of the geometry of the constellation. If we are interested in avoiding large variations of the mutual distances between the spacecraft, we can consider the possible existence of regions of zero relative radial acceleration with respect to the nominal trajectory. The motion along these regions will reduce the expansion or contraction of the constellation. The goal of this paper is the study of these regions and the controlled motions between them.

1 INTRODUCTION

Over the last years, the use of constellations of spacecraft has gained much attention among mission planners because of its many applications, such as multi–spacecraft interferometry. We want to consider guidance and control strategies for these kind of applications, but making abstraction of technological issues, so we will assume that any spacecraft is able to perform any maneuver in any direction. In this setting, many efforts have been done up to now, like the constellation–reorientation algorithms with fuel–balancing of R.W. Beard, T.W. McLain and F.Y. Hadaegh (see [1, 2, 3, 4, 5]), and the use of artificial potential functions of C.R. McInnes (see [9, 11, 10, 12]) for collision avoidance purposes.

In the present paper, we want to study natural configurations suitable for formation flying, as well as controlled motions between these configurations. These configurations will be based on dynamical and geometrical

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considerations, in order to make them independent of the force model under consideration. Most of the work is done in the Restricted Three–Body Problem, but all the strategies developed can be easily extended to the general n–body problem, as is shown in the paper.

The concrete goals of the paper are:

- The study of geometries, around arbitrary nominal orbits of the n–body problem, with good properties for formation flight.
- The study of controlled motions between the zero relative radial acceleration cones (ZRRAC) obtained from the preceding analysis.

In all the simulations that follow, we will use one of the following four reference trajectories:

- A halo orbit of moderate amplitude (150 000 km) around the L_2 point of the Sun–(Earth+Moon) RTBP (see Fig. 1). It has been computed by a Lindstedt–Poincaré procedure [6, 8].
- A transfer trajectory to the previous halo orbit, taken in the branch of its stable manifold that approaches the Earth (see Fig. 1).
- Refined versions of the previous two trajectories, in the solar system model given by the JPL DE403 ephemeris, taking Jan 1, 2000 as starting epoch. They have been obtained from the RTBP trajectories by a multiple–shooting procedure [7].

Figure 1: Halo orbit of the RTBP, transfer trajectory and coordinate projections of the two previous trajectories. The points marked with a cross show the position of the Earth and its projections on the coordinate planes.

2 THE ZERO RELATIVE RADIAL ACCELERATION CONES

In order to avoid expansion or contraction in a constellation of spacecraft, with the corresponding large variations of the mutual distances between them, we have studied the existence of regions with zero relative radial acceleration (ZRRA). For a simple model, such as the RTBP, it is possible to get an analytical expression for the above regions, provided the radius of the constellation (largest separation between the spacecraft) is small, so that a linear approach to the problem gives the relevant information about the local dynamics of the problem.

We write the RTBP equations of motion as the first order system of differential equations [13],

$$
\begin{array}{rcl}
\dot{x} &=& \xi, \\
\dot{y} &=& \eta, \\
\dot{z} &=& \zeta, \\
\dot{\xi} &=& 2\eta + x - (x-\mu)\dfrac{1-\mu}{r_1^3} - (x-\mu+1)\dfrac{\mu}{r_2^3}, \\
\dot{\eta} &=& -2\xi + y\left(1 - \dfrac{1-\mu}{r_1^3} - \dfrac{\mu}{r_2^3}\right), \\
\dot{\zeta} &=& -z\left(\dfrac{1-\mu}{r_1^3} + \dfrac{\mu}{r_2^3}\right),
\end{array}
\tag{1}
$$

with $r_1 = \sqrt{(x-\mu)^2 + y^2 + z^2}$, $r_2 = \sqrt{(x-\mu+1)^2 + y^2 + z^2}$.

Denoting the above system by $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, the linear behavior around a solution $\mathbf{x}(t)$ is given by

$$\dot{\mathbf{u}} = D\mathbf{f}(\mathbf{x}(t))\mathbf{u}, \tag{2}$$

where

$$
Df = \left(\begin{array}{ccc|ccc}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\hline
f_x^4 & f_y^4 & f_z^4 & 0 & 2 & 0 \\
f_x^5 & f_y^5 & f_z^5 & -2 & 0 & 0 \\
f_x^6 & f_y^6 & f_z^6 & 0 & 0 & 0
\end{array}\right) = \left(\begin{array}{c|c} 0 & I \\ \hline F & J \end{array}\right),
$$

and f^4, f^5, f^6 are the last three component of the vector-field $\mathbf{f}$, of which we have to compute their partial derivatives with respect to x, y and z in order to get the symmetric sub–matrix F. Writing the array $\mathbf{u}$ as $(\mathbf{r}, \dot{\mathbf{r}})^\top$, the linear system of Eq. (2) becomes

$$
\left(\begin{array}{c} \dot{\mathbf{r}} \\ \ddot{\mathbf{r}} \end{array}\right) = \left(\begin{array}{c|c} 0 & I \\ \hline F & J \end{array}\right) \left(\begin{array}{c} \mathbf{r} \\ \dot{\mathbf{r}} \end{array}\right). \tag{3}
$$

The points with zero relative velocity are those such that $\dot{\mathbf{r}} = 0$, and, in this case, we have that the relative acceleration is given by

$$\ddot{\mathbf{r}} = \dot{\mathbf{v}} = F\mathbf{r}.$$

Figure 2: Zero relative radial acceleration cones along the reference halo orbit of the RTBP.

To get the radial component of the relative acceleration we must compute the scalar product of $\ddot{\mathbf{r}}$ with $\mathbf{r}$. This radial component will be zero for the set of points such that

$$\mathbf{r}^\top F\,\mathbf{r} = 0. \tag{4}$$

Eq. (4) represents, in general, a quadric which depends on the point $\mathbf{x}(t)$ selected along the nominal solution of Eq. (1). For the reference halo orbit displayed in Fig. 1, we obtain zero relative radial acceleration cones (ZRRAC), which are shown in Fig. 2 at different points of the orbit.

The ZRRAC can be also computed numerically as follows. Given a certain nominal trajectory, we selected a point on it $(\mathbf{x}(t), \mathbf{v}(t))$. Around this point we consider a sphere, in configuration space, of radius equal to 3×10^{-9} dimensionless RTBP units (0.5 km), and we set the velocity of all the points of the sphere equal to the velocity of the point selected, that is, $\mathbf{v}(t)$ (zero relative velocity condition). Parameterizing the sphere by the longitude λ and the latitude ϕ, the test points of the sphere will be of the form

$$(\mathbf{x}(t) + R\mathbf{s}(\lambda, \phi), \mathbf{v}(t)),$$

with $R = 0.5\,\mathrm{km}$ and $\|\mathbf{s}(\lambda, \phi)\| = 1$. Now, writing the equations of motion of the RTBP as

$$\ddot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, \dot{\mathbf{x}}),$$

we can evaluate the relative acceleration by

$$\mathbf{a}_r(t; \lambda, \phi) = \mathbf{g}\big(\mathbf{x}(t) + R\mathbf{s}(\lambda, \phi), \mathbf{v}(t)\big) - \mathbf{g}(\mathbf{x}(t), \mathbf{v}(t)),$$

whose scalar product with $\mathbf{s}(\lambda, \phi)$ will give the desired relative radial acceleration.

$$a_{rr}(t; \lambda, \phi) = \langle \mathbf{a}_r(t; \lambda, \phi), \mathbf{s}(\lambda, \phi) \rangle \tag{5}$$

In Fig. 3 we show the behavior of this function (in red) for different points along the reference transfer trajectory of the RTBP. (Each plot corresponds to a different point on the orbit, i.e., to a different value of t in Eq. (5)). In each plot of this figure, we have also displayed (in magenta) the corresponding ZRRA plane. The intersection of this plane with the RRA surfaces corresponds to the intersection of the ZRRAC of vertex $\mathbf{x}(t)$ with the sphere of radius 0.5 km around $\mathbf{x}(t)$.

Figure 3: Relative radial velocity surfaces associated to several points along the transfer trajectory of the RTBP.

The qualitative behavior of the function $\mathbf{a}(t; \lambda, \phi)$ is almost the same for all the values of t, either if we move along the halo orbit or along the transfer trajectory. There appear two maxima, associated to the unstable directions, and two minima, related to the stable ones. The function $\mathbf{a}$ is zero along two cones with vertex at $\mathbf{x}(t)$. These cones give the most suitable directions to set a constellation of spacecraft. If these cones were invariant by the dynamics, a set of aligned spacecraft placed on them would keep fixed their mutual distances. Actually, these distances will vary but this variation will be slow. This relative behavior will be shown later.

The cones of zero relative radial acceleration obtained with the analytical linear approach reproduce, qualitatively and quantitatively, the behavior detected numerically. The numerical determination can be easily extended to more realistic models of motion, such as a model of the Solar System based on the JPL DE403 ephemeris file, as shown in Fig. 4.

Figure 4: Relative radial velocity surfaces associated to four different points of the refined halo orbit in the full JPL Solar System model.

3 DYNAMICAL BEHAVIOR ALONG ZRRAC

In this section we show the dynamical behavior of the different kinds of solutions with initial conditions along the most relevant directions determined in the preceding section.

For the first simulation, we have taken five points $p_0, \dots, p_4$ along the direction of maximum radial acceleration associated to the initial condition of the halo orbit of the RTBP. Of these points, p_0 is the initial condition, and the remaining ones are distributed symmetrically with respect to p_0: p_2, p_3 being at a distance of 0.25 km from p_0, and p_1, p_4 at a distance of 0.5 km.

The results of a numerical integration of these points for a full period of the halo orbit (approximately 180 days) are shown in Fig. 5. As it can be seen, even starting along the most expansive direction, the orbits do not deviate significantly from the periodic orbit during this time span. Nevertheless, the behavior of the distance to the halo orbit, $d(p_i(t), p_0(t))$, is exponential, as it should be (Fig. 5 right). Also, there is no difference between the qualitative behavior of $d(p_i(t), p_0(t))$ for the trajectories starting at the same distance from p_0: $d(p_2(t), p_0(t)) \approx d(p_3(t), p_0(t))$ and $d(p_1(t), p_0(t)) \approx d(p_4(t), p_0(t))$.

Figure 5: Left: trajectories followed by the points chosen along the direction of maximum relative radial acceleration. Right: distances, $d(p_i(t), p_0(t))$, between the trajectories of p_1, p_2, p_3, p_4 and the base halo orbit $p_0(t)$.

Figure 6: Values of the differences $d(p_i(t), p_0(t)) - d(p_i(0), p_0(0))$ ($d(p_i(0), p_0(0)) = 0.25$ km for $i = 2, 3$ and 0.50 km for points $i = 1, 4$) for a 5–day time span.

In Fig. 6 we have displayed the deviations of the actual positions with respect to the initial ones: $d(p_i(t), p_0(t)) - d(p_i(0), p_0(0))$. As it can be seen from this figure, after five days they are of the order of 14 meters. This corresponds to an acceleration of $1.5 \times 10^{-10}\,\mathrm{m/s^2}$, which is approximately equal to the maximum relative radial acceleration computed along this orbit (see Fig. 4).

Now, let us test the opposite situation and start from four points along a ZRRAC generatrix, corresponding also to the initial condition of the halo orbit of the RTBP, distributed in a similar fashion to the previous case. We will denote these new points as $q_i(t)$, $i = 0, \ldots, 4$. In Fig. 7 we show the results for the $d(q_i(t), q_0(t))$ function, corresponding to the integration during a full period (180 days) of the different initial conditions. Now, although the qualitative behavior is still exponential, the final distances are shorter (they are reduced by a factor of 3) than the ones obtained for the p_i points taken along the unstable direction.

In the left plot of Fig. 8 we display the results corresponding to an integration analogous to the one of Fig. 6. As it can be seen, the maximum deviation from the starting separations is now less than 50 cm while for the p_i points was of 14 m. In the right plot of the same figure, we represent the separations for a 50–day time–interval. We can observe a change of behavior of the relative distance, from being governed by radial accelerations around the base orbit to being governed by the exponential escape inherent to the libration point orbit.

In order to show that the JPL model behaves in the same way as the RTBP does, we have performed the same two kinds of computations as before: taking initial conditions along the “worst” and “best” directions, using Jan 1, 2000 as initial epoch. In Figs. 9 and 10 we have represented the deviation from the initial separation from the base orbit of the points p_i and q_i of the JPL model. It is seen that the behavior is very similar to the one displayed for the RTBP model.

4 CONTROLLED MOTIONS CONNECTING ZRRAC

In this section we will show some results related to the control of a formation moving within zero relative radial acceleration cones. For simplicity, we will assume that the formation has only three spacecraft: two of them at the edges of a segment and the third one at the middle point. This third spacecraft will move along the reference halo orbit without any control acting on it. The two edge spacecraft will be controlled by a bang–bang procedure.

In the first situation considered, we fix a point on the reference halo

Figure 7: Distances, $d(q_i(t), q_0(t))$, between the trajectories of q_1, q_2, q_3, q_4 and the base halo orbit $q_0(t)$.

Figure 8: Deviations $d(q_i(t), q_0(t)) - d(q_i(0), q_0(0))$ from the starting separations from the nominal orbit of the paths followed by the points q_1, q_2, q_3, q_4, for a 5–day time–interval (left) and for a 50–day one (right).

Figure 9: Deviations from the starting separations from the nominal orbit of the p_i points (left) and the q_i points (right) for a 180–day interval in the JPL model.

orbit of the RTBP, put a spacecraft in it, and symmetrically distribute two more spacecraft on a generatrix of the associated cone. After Δt time units, during which the central spacecraft moves along the orbit, the other two are controlled to be on a segment parallel to the initial one and keeping their mutual distances fixed.

This situation is illustrated in Fig. 11. We have used different values for the time displacement of the central spacecraft along the reference orbit: $\Delta t = 2, 3, 4$ and 5 days. For any of these values, we have computed the impulsive maneuvers that set the spacecraft at the corresponding edges of the segment in the same amount of time. It must be noted that once the formation has moved from its initial position, the segment that contains it is not, in general, on any generatrix of any zero radial acceleration cone. That is, the ZRRAC are not invariant by the dynamics.

If $\mathbf{x}_i$, $\mathbf{x}_f$ represent the initial and final states (position and velocity) of a spacecraft, $\Delta\mathbf{v}_1$, $\Delta\mathbf{v}_2$ the maneuvers to be applied and ϕ_t the time–t flow of the RTBP, the equations that must be solved for the computation of the impulsive translation maneuvers are

$$\phi_{(1-\alpha)\Delta t}\left[\phi_{\alpha\Delta t}\left(\mathbf{x}_i + \begin{pmatrix} 0 \\ \Delta\mathbf{v}_1 \end{pmatrix}\right) + \begin{pmatrix} 0 \\ \Delta\mathbf{v}_2 \end{pmatrix}\right] = \mathbf{x}_f.$$

Note that this is a system of six equations with seven unknowns: the components of the two impulses and the parameter α. We have used the value of α that minimizes $\|\Delta\mathbf{v}_1\| + \|\Delta\mathbf{v}_2\|$, although there is not a significantly variation of this magnitude with α.

Fig. 12 shows the total cost (cm/s) of the parallel translation maneuvers for the two spacecraft, for $\Delta t = 2, 3, 4$ and 5 days, when the vertex of the departure cone moves along the halo orbit and the distance between the spacecraft at the edges of the segment is of 1 km. Only one generatrix has been taken on each cone. The Δv required for the maneuvers of each spacecraft is, approximately, one half of the total cost. For any value of Δt there is a point on the halo orbit at which the cost of the translation is maximum. This point corresponds to the lower point of the halo orbit, which is the point at which the gravitational influence of the Earth is larger

Figure 10: Deviations from the starting separations from the nominal orbit of the p_i points (left) and the q_i points (right) for a 5–day interval in the JPL model.

Figure 11: After some Δt, the spacecraft are controlled to be on a line parallel to the initial configuration, which is on a generatrix of a zero radial acceleration cone.

and the cost of the transfer at this (or any other) point behaves almost linearly with Δt.

Next we have fixed a departure cone and we have looked for the minimum and maximum cost generatrices of this cone. We have parameterized the ZRRAC generatrices by an angle in radians (its zero value does not have any special meaning). The results corresponding to take as departure cone the ZRRAC of the initial condition of the reference halo orbit are given in Fig. 13. As it is clearly seen, for any value of the Δt displacement, there are two angles on the cone for which the cost of the transfer is minimum and two other values for which is maximum. The angles, along the departure cone, corresponding to these four situations are, approximately, equal to 0, $\pi/2$, π, $3\pi/2$ and 2π. These four directions will be used later.

For the second kind of explorations, both the initial and final configurations of the formation are on a generatrix of a zero radial acceleration cone, so the transfer is non-parallel.

In the first exploration, the departure generatrix is fixed and the arrival one moves along the arrival cone, at a distance Δt from the first. The results corresponding to these transfers are given in Fig. 14. When the initial configuration is almost parallel to the final one (both with angle along the cone equal to zero) the cost of the transfer is minimum, independently of the value of Δt. This pattern of behavior of the cost function is independent of the position of the initial configuration along the halo orbit, as is shown in Fig. 14 (right). From this figure one can see that the cost decreases as Δt increases. This is true for almost all the values of angle of the arrival generatrix except for those close to zero, for which the situation is reversed, according to Fig. 12.

Next, we have taken as departure generatrices those which correspond to the minimum and maximum values of the cost function from the parallel transfers. The results are given in Fig. 15 for $\Delta t = 2$ and 5 days.

Finally, we have studied the transfers from an arbitrary generatrix to an arbitrary generatrix of two fixed cones. The results are shown in Fig. 16. From this figure is clear that the cost surfaces reach their minima on the diagonal of the x–y plane. This means that the transfer costs are minimum

Figure 12: From bottom to top, the different curves represent the total cost (cm/s) of the parallel translation maneuvers for $\Delta t = 2, 3, 4$ and 5 days, when the vertex of the departure cone moves along the halo orbit. The distance between spacecraft at the edges of the segment is of 1 km. Only one generatrix has been taken on each cone.

Figure 13: Behavior of the parallel translation cost (cm/s) as a function of the angle of the generatrix on the departure cone. The curves correspond to different values of $\Delta t = 2, 3, 4, 5$ (from bottom to top, respectively). The vertex of the cone has been kept fixed.

when both the initial and final generatrix are almost parallel.

5 FORMATION FLYING USING ZRRAC

From the simulations of the previous section, we can conclude that, for the three–spacecraft formation described there, the minimum–cost transfers from a ZRRAC generatrix to a ZRRAC generatrix are obtained when the departure and arrival generatrices are parameterized by the same angle.

In this section, we will show the results of several simulations of a three–spacecraft formation like the one of the previous section. These three spacecraft will be controlled so that, at prescribed epochs, the center s/c is on the reference transfer trajectory of the RTBP, and the edge ones are on a generatrix of the corresponding ZRRAC, at distances d_1 and d_2 of the central s/c. All the generatrices will be taken as parameterized by the same angle α_0. For the control, we will use the procedure that is described next.

5.1 The minimum Δv control strategy

This control procedure solves the following basic problem: consider a nominal path, defined by a certain initial state

$$(t_0, x_0, v_0),$$

and a true state of the spacecraft at $t = t_0$, given by (see Fig. 17)

$$(t_0, x_0 + \Delta x, v_0 + \Delta v) = (t_0, x_t, v_t).$$

The goal is to recover the nominal path at a certain epoch $t_N > t_0$, this is, we want to reach the state

$$\phi_{t_N - t_0}(x_0, v_0),$$

Figure 14: Total cost (cm/s) of the non-parallel translation maneuvers between two ZRRAC separated $\Delta t = 2, 3, 4$ and 5 days (top to bottom curves and surfaces). In the left figure, the departure configuration is fixed on a generatrix with angle equal to zero and the final one moves along the arrival cone, that has been parameterized by an angle varying between 0 and 2π represented on the x axis of the figure. In the right hand side figure the initial point moves along the halo orbit (x–axis measured in dimensionless time).

Figure 15: Total cost (cm/s) of the non-parallel transfers from the minimum (m1 and m2 curves) and maximum (M1 and M2 curves) transfer cost directions of the departure cone. The left figure corresponds to $\Delta t = 2$ and the figure on the right to $\Delta t = 5$ days.

where ϕ is the flow associated to the problem. The solution to this basic problem can be easily adapted to the case in which the final state of the spacecraft, at $t = t_N$, is not $\phi_{t_N - t_0}(x_0, v_0)$ but some well defined state: $\phi_{t_N - t_0}(x_0, v_0) + (\Delta x_N, \Delta v_N)$.

This control problem has been solved as follows: we introduce a sequence of maneuvers

$$\Delta v_0, \Delta v_1, ..., \Delta v_N,$$

to be done at some chosen epochs

$$t_0, t_1, ..., t_N.$$

The maneuvers should then verify the following constraint:

$$\phi_{t_N - t_{N-1}}\left(\ldots \phi_{t_2 - t_1}\left(\phi_{t_1 - t_0}(x_t, v_t + \Delta v_0) + \Delta v_1\right) + \cdots + \Delta v_{N-1}\right) + \Delta v_N = \phi_{t_N - t_0}(x_0, v_0).$$

Of course, there are infinitely many different values of $\Delta v_0, \Delta v_1, ..., \Delta v_N$ verifying the above equation. The ones selected minimize

$$\sum_{j=0}^{N} q_j \|\Delta v_j\|^2,$$

where q_0,...,q_N are weights which must be fixed in advance. For the simulations we have used

$$q_j = 2^{-j},$$

so the magnitude of two consecutive maneuvers decays approximately by a factor of 2. For the solution of this problem, the flow ϕ can be replaced by its linear approximation, given by the variational equations, provided we are not far from the nominal path.

Figure 16: Total cost (cm/s) of the cone to cone transfers for $\Delta t = 2$ (lower surface) and 5 days (upper surface) as a function of the departure and arrival angles of the generatrices on their respective cones.

Figure 17: Illustration of the control procedure.

5.2 Simulation parameters and summary of results

The basic parameters of the simulations are:

The tracking time interval: T_t. After T_t time units of uncontrolled flight, the formation is tracked and a control maneuver is started.

The time interval of a control maneuver: $T_m(= t_N - t_0)$. Each control maneuver is composed by several correction maneuvers, and lasts T_m time units. At the end of this time interval the formation is recovered, so that the three s/c are on a ZRRAC generatrix every $T_t + T_m$ time units (see Fig. 18). The length of the control maneuver interval must be such that $T_m \leq T_t$. In the simulation program, during this time interval all the correction maneuvers are executed at uniformly distributed instants ($t_{i+1} - t_i =$ constant). This choice can be easily modified.

The number of correction maneuvers of each control: N. In order that each spacecraft can recover its position in the formation, at least two correction maneuvers must be done (assuming that they are performed without errors). We have foreseen the execution of $N \geq 2$ correction maneuvers with decreasing magnitude (the decay will be approximately as $1/2^n$). A typical plot displaying this kind of behavior, for the magnitude of the correction maneuvers as a function of time, is shown in Fig. 19.

Figure 18: Illustration of the meaning of the parameters of the simulations.

We have done several simulations in order to see the behavior of the formation, using different sets of values of the parameters marked as "free" in the following table.

	Angle α_0	Distance d_1	Distance d_2	Tracking interval	Number of maneuvers	Execution time interval
Simulation 1	Fixed	Fixed	Fixed	**Free**	Fixed	Fixed
Simulation 2	Fixed	Fixed	Fixed	Fixed	Fixed	**Free**
Simulation 3	Fixed	Fixed	Fixed	Fixed	**Free**	Fixed
Simulation 4	Fixed	**Free**	**Free**	Fixed	Fixed	Fixed
Simulation 5	**Free**	Fixed	Fixed	Fixed	Fixed	Fixed

Figure 19: Magnitude of the correction maneuvers as a function of time. Each control maneuver is composed by 4 correction maneuvers with decreasing magnitude.

For every set of values of the parameters considered, we have analyzed the behavior of several variables:

1. For each spacecraft of the formation, the x, y and z components of the differences between the actual position of the spacecraft and its nominal one. These differences are shown as a function of time. A typical result, for the x component, is displayed in the following figure.

   Each column corresponds to a different spacecraft. Since, for this simulation, the two edge spacecraft of the formation are at the same distance from the central one, their corresponding figures are almost symmetrical. The deviation of the central spacecraft with respect to its nominal position is almost negligible, since we require this s/c to follow a true trajectory of the model problem (the RTBP). Each time a correction maneuver is applied, the deviation decreases and it is almost zero after the last correction maneuver of a given control maneuver.

2. For each spacecraft of the formation, the magnitude of the control maneuvers applied. An example is shown in the next figure. Of course, the magnitude of the controls are closely related to the deviations, so the pattern of these figures is close to the previous ones.

3. For each spacecraft of the formation, the total magnitude of the controls applied (which is the sum of the magnitudes of all the control maneuvers. The following figure is an example.

The following items summarize the basic results obtained:

1. The best results are obtained for a spacecraft at $d = 0.5$ km, with a tracking time interval equal to the execution of maneuvers time interval: $T_t = T_M = 5$ hours. During this interval $N = 5$ maneuvers are executed. The total Δv required for the transfer of this spacecraft, keeping its formation configuration, is approximately equal to 14.2 cm/s.

2. In the above case, if the distance d is doubled, the total Δv is also multiplied by 2.

3. For the simulations done varying the tracking time interval and keeping fixed the remaining parameters, it has been found that:

   (a) The maximum deviation increases almost linearly with the tracking interval.

   (b) The maximum Δv increases also almost linearly with the tracking interval.

   (c) For a large set of values of the tracking interval, the total Δv is almost constant. When the tracking interval is less than 4 hours, the Δv decreases abruptly and can be reduced even to a 70% of its almost constant value.

4. For the simulations done varying the execution of maneuvers time interval and keeping fixed the remaining parameters, it has been found that:

   (a) The maximum deviations seem to be independent of the free parameter and remain almost constant.

   (b) There is an almost exponential decrease of both the maximum and the total Δv when the time interval for the execution of maneuvers increases and gets close to the length of the tracking interval.

5. For the simulations done varying the number of maneuvers, it has been found that:

   (a) The maximum deviations seem to be independent of the free parameter and remain almost constant.

   (b) Both the maximum and the total Δv increase almost linearly with the number of maneuvers. The best values are obtained when only two maneuvers are executed.

6. For the simulations done varying the separation between the spacecraft, it has been found that:

   (a) When the separation increases, the maximum deviations also do in an almost linear way. This is true up to separations of the order of 3 km, which are probably at the boundary of the linear approximations used.

   (b) Both the maximum and the total Δv increase almost linearly with the separation between the spacecraft. The best values are obtained when this separation is very small.

7. For the simulations done varying the angle α_0 in the cone of zero relative radial acceleration, it has been found that:
   (a) The smallest deviations are found for values of the angle between 60 and 120 degrees.
   (b) Close to $\alpha_0 = 60^\circ$ and $\alpha_0 = 300^\circ$ there are two local minima of the total Δv.

6 CONCLUSION

In this paper we have introduced some new manifolds associated to any natural trajectory suitable for formation flight. These manifolds are not invariant under the dynamics. Nevertheless, the natural motion using initial conditions for the spacecraft of the formation on these manifolds avoids large variations of the mutual distances between the spacecraft. Different kinds of controlled motions between these manifolds have been analyzed.

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